

Kenmerk: EWI2021/TW/DMMP/MU/

Test Exam 1, Module 7 Discrete Structures & Efficient Algorithms

All answers need to be motivated. No calculators. You are allowed to use a handwritten cheat sheet (A4, both sides) per topic (ADS, DM).

This exam consists of two parts, with the following (estimated) times per part:

Algorithms & Data Structures (ADS)	1h	(30 points)
Discrete Mathematics (DM)	2h	(60 points)

Total of $30+60=90$ points. Your exam grade equals $1 + 0.1x$ rounded to one digit, with x being your number of points. That means 45 points to pass.

Please use a new sheet of paper for each part (ADS, DM)!

Algorithms & Data Structures

See other pdf's for example questions for the ADS part.

Discrete Mathematics

4. (10 points)

- (a) Show that the Diophantine equation $1000s + 444t = 2$ has no solution for $s, t \in \mathbb{Z}$.
- (b) Show that for $a, b, c \in \mathbb{Z}$, if $\gcd(a, b) \mid c$, then equation $as + bt = c$ has a solution in integers (s, t) .

5. (10 points)

- (a) Compute the solution to the recurrence relation

$$a_n - 10a_{n-1} + 21a_{n-2} = 60 \cdot 3^n \quad (n \geq 2) \quad \text{with} \quad a_0 = 2 \text{ en } a_1 = -5.$$

- (b) Consider strings in $\{0, 1, 2\}^*$. Let a_n be the number of strings in $\{0, 1, 2\}^*$ of length n that do not contain the substring 01 and neither 02. Compute a_1, a_2, a_3 and a recurrence relation for a_n for all $n \geq 4$. (You do not need to solve this recurrence relation.)

6. (10 points) Let $G = (V, E)$ be a simple, undirected graph, with edge weights $d_e > 0$, $e \in E$. Let $T \subseteq E$ be a minimum weight spanning tree (MST) for G . Moreover, for a fixed node $s \in V$, let us denote by $D(s, v)$ all edges that lie on shortest (s, v) -paths, for $v \in V$. Let $D(s) := \bigcup_{v \in V} D(s, v)$. Prove that $T \cap D(s) \neq \emptyset$.
7. (10 points) Let $G = (V, E)$ be a simple, bipartite undirected graph. Let $|V| = n$ and $|E| = m > 1$. Prove or give a counterexample:
- If $m \leq 2n - 4$, Then G is planar.
 - If G is planar, then $m \leq 2n - 4$.
8. (10 points) How many possibilities are there to distribute 50€ over three persons, such that nobody gets less than 10€, and somebody gets at most 15€? Use a generating function to compute the answer.
9. (10 points) Give a short proof or give a counterexample for each of the following statements.
- Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$, $e \in E$. Then any two minimum spanning trees T_1 and T_2 for G must have a nonempty intersection, that is, $T_1 \cap T_2 \neq \emptyset$.
 - Consider a capacitated network $G = (V, A, c)$, where V is the set of vertices, A is the set of directed arcs, and $c_a \geq 0$, $a \in A$ are the arc capacities. Then there always exists a maximum flow f_a , $a \in A$, such that either $f_a = 0$ or $f_a = c_a$ for all $a \in A$.
 - Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$ such that $w_e \neq w_{e'}$ for all $e, e' \in E$, $e \neq e'$. Let $s \in V$ be fixed. Then for all $v \in V$ there is a unique shortest (s, v) -path.
 - Consider an undirected, simple graph $G = (V, E)$ with edge weights $w_e \geq 0$ such that $w_e \neq w_{e'}$ for all $e, e' \in E$, $e \neq e'$. Then there is a unique minimum spanning tree T .