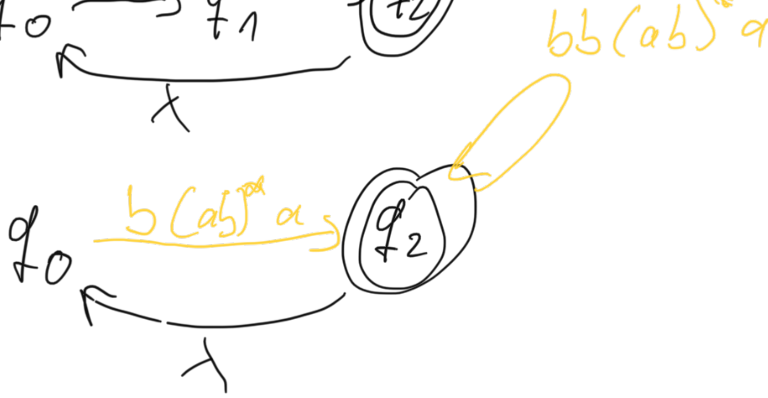
LFM 2019

1 a)

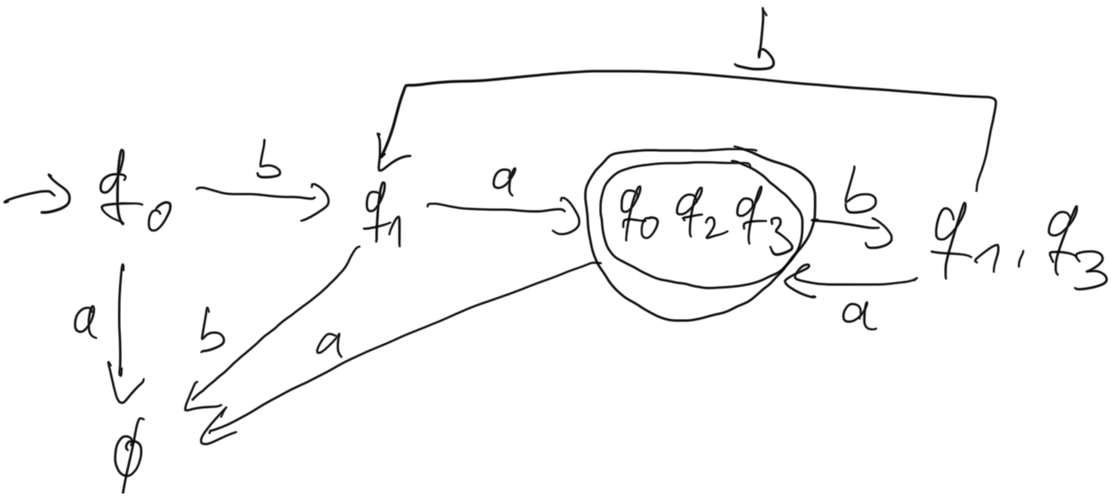
b)



$$\left[b(ab)^*a\,(bb(ab)^*a)^*\right]^+$$

c)

	a	b	λ-cl.
q_0	$\emptyset$	q_1	q_0
q_1	q_2, q_0, q_3	$\emptyset$	q_1
q_2	$\emptyset$	q_1, q_3	q_2, q_0
q_3	$\emptyset$	q_1	q_3



2. a) regular: $L_1 = L((aa)^* b^*)$

b) not regular.

Proof using Pumping Lemma

Let k be given. Choose $z = a^k b a^k$.

Then $z \in L_2$ and $|z| \geq k$.

Let $u\,v\,w$ be given with $z = uvw$ and $|uv| \leq$

and $|v| > 0$.

Then $u = a^p$ and $v = a^q$ with $p+q \leq k$

and $w = a^{k-p-q} b a^k$

choose $i = 0$, then $uv^0w = a^{k-q} b a^k \notin$

So L_2 is non-regular.

c) L_3 is regular as every finite language is regular. As reg. languages are closed under complement and reversal, $\overline{L_3}$ and L_3^R are reg. as well.

The union of ~~two~~ reg. lang. is regular, hence $\overline{L_3} \cup L_3^R$ is reg.

3.)

$chain(C) = \{A, B, C\}$

$$G_1' = \begin{cases} S \to AB \mid cC \mid a \mid aA \mid bB \\ A \to aA \mid bB \mid cC \mid a \\ B \to bB \mid cC \mid a \mid aA \\ C \to cC \mid a \mid aA \mid bB \end{cases}$$

4.)

$S \to bS \mid b \mid aA'$

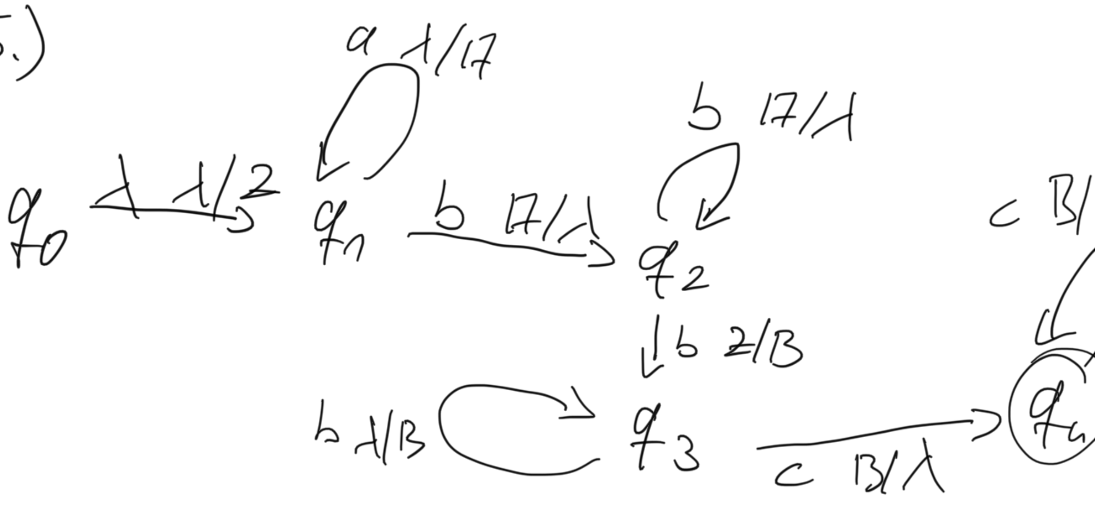
$A' \to aB \mid aA''$

$A'' \to aA'$

$B \to b$

$b^*(aa)^*b$

5.)



- add symbol Z ~~for~~ bottom of stack
- In q_1 count #a on stack
- to end in q_2 count downwards on

stack until #b is equal to number of previous a's.

- for the remaing b's add B's to stack
- In and towards q_F count #c such that stack is empty if and only if #c is equal number of additional b's

6.) $L(P) \cap L(E)$ is context-free since it can be described by a PDA as the product of the DPDA P and a DFA M with $L(M) = L(E)$.

$L(G)$ is context-free.

The class of context-free languages is closed under union. thus $L(G) \cup (L(P) \cap L(E))$ is also context-free.

7. Let L be an arbitrary recursive language and $M = (Q, \Sigma, \Gamma, \delta, q_0, F)$ a det. TM

<u>decides</u> L:

For all $w \in \Sigma^*$: M started with input w

halts in an accepting state $q \in F$
For all $w \notin L$: M started with input w halt in state $q \notin F$.

Let $M' = (Q, \Sigma, \Gamma, q_0, F')$ with $F' = Q \setminus F$

Then, for every $w \in L$: M' started with input w halt in a state $q \notin F'$
For every $w \notin L$, M' halts in a state $q \in F'$. Thus $L(M') = \Sigma^* \setminus L(M)$
□

8.)
$L = \{ w w^R \mid w \in \{a,b\}^+ \}$

After reading the leading blank ($q_0 \to q_1$)
The TM replaces the first and the last a with an X ($q_1 \to q_2 \to q_4 \to q_6$)
or the first and last b with an X
($q_1 \to q_3 \to q_5 \to q_6$).
It then goes back to the beginning of the word (not counting X) (loop in q_6) and repeats this process.
If there is is no first a or b that can be replaced and if there is at least

[illegible] least

one X, the TM accepts.