Lecture L&M 8

Unrestricted Grammars, Undecidability

8.1 Unrestricted grammars

A. Give an unrestricted grammar that generates the language $\{a^i b^j a^i b^j \mid i, j \geq 0\}$.

B. *(extra)* Give an unrestricted grammar that generates the language $\{a^i b^i c^i d^i \mid i \geq 0\}$.

C. Prove that the language generated by the following grammar is $\{a^i b^i c^i \mid i \geq 0\}$:

$$
\begin{aligned}
S &\to aAbc \mid \lambda \\
A &\to aAbC \mid \lambda \\
Cb &\to bC \\
Cc &\to cc
\end{aligned}
$$

8.2 Undecidability

D. The *n-step halting problem* is as follows: Given a Turing machine M and a word w, decide whether M halts after performing at most n transitions when run on input w.

Is this problem decidable?

E. Let L be a (fixed) language that is recursively enumerable but not recursive. Let M be a (fixed) Turing machine that recognises L. The *halting problem for M* is defined as follows: Given a word w, decide whether M halts on input w.

Prove that this problem is undecidable (for any L and M).

F. Consider the following potential argument for the undecidability of the blank tape problem:

The blank tape problem is a subproblem of the halting problem,
which is undecidable and therefore must be undecidable itself.

Why is this *not* a valid argument?

G. Prove that the *101 halting problem* of deciding whether an arbitrary given Turing machine halts when run on input 101 is undecidable. Use a reduction.