

Lecture L&M 7

Pumping Lemma for Context-Free Languages, Turing Machines

7.1 Pumping lemma for context-free languages

For each of the following languages L , use the pumping lemma to prove that it is not context-free:

A. $L = \{a^k \mid \exists n \in \mathbb{N}: k = n^2\}$

B. $L = \{a^i b^j c^i d^j \mid i, j \geq 0\}$

C. (extra) $L = \{a^i b^{2i} a^i \mid i \geq 0\}$

D. (extra extra) L is the set of finite-length prefixes of the infinite word

$$abaabaabaabaab \dots ba^n ba^{n+1} b \dots$$

7.2 Standard Turing machines

For each of the following languages, give a standard Turing machine that recognises it.

E. Language L from Exercise B.

F. Language L from Exercise D.

7.3 Multi-tape Turing machines

For each of the following languages, give a multi-tape Turing machine that recognises it. The input tape (tape 1) must only be processed from left to right (i.e. only the directions S and R , but not L , are allowed for tape 1).

G. (extra) Language L from Exercise B.

H. (extra) Language L from Exercise D.

7.4 Two-way Turing machines

- I. Construct a two-way Turing machine with input alphabet $\{a\}$ that accepts an input when it is of the form a^{2n+1} for some $n \geq 0$, but only if the read head is initially on the middle symbol.

7.5 Non-deterministic Turing machines

- J. Construct a (possibly nondeterministic) two-tape Turing machine that decides the language $L = \{uu \mid u \in \{a, b\}^*\}$ such that every computation on input w with $n = |w| \geq 3$ terminates after at most $2n + 3$ transitions.