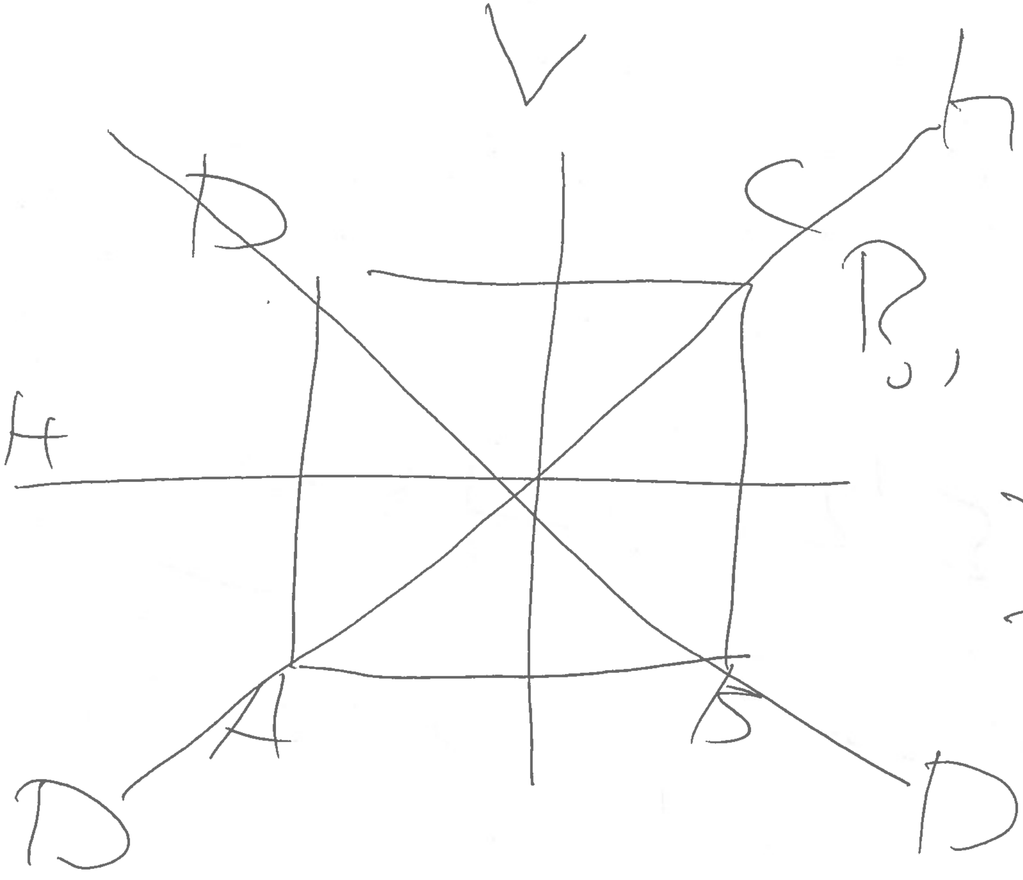
Rule 1: please be in time

2: one person at a time may speak



$R_0, R_{90}, R_{180}, R_{270}, \cancel{R_{360}}$

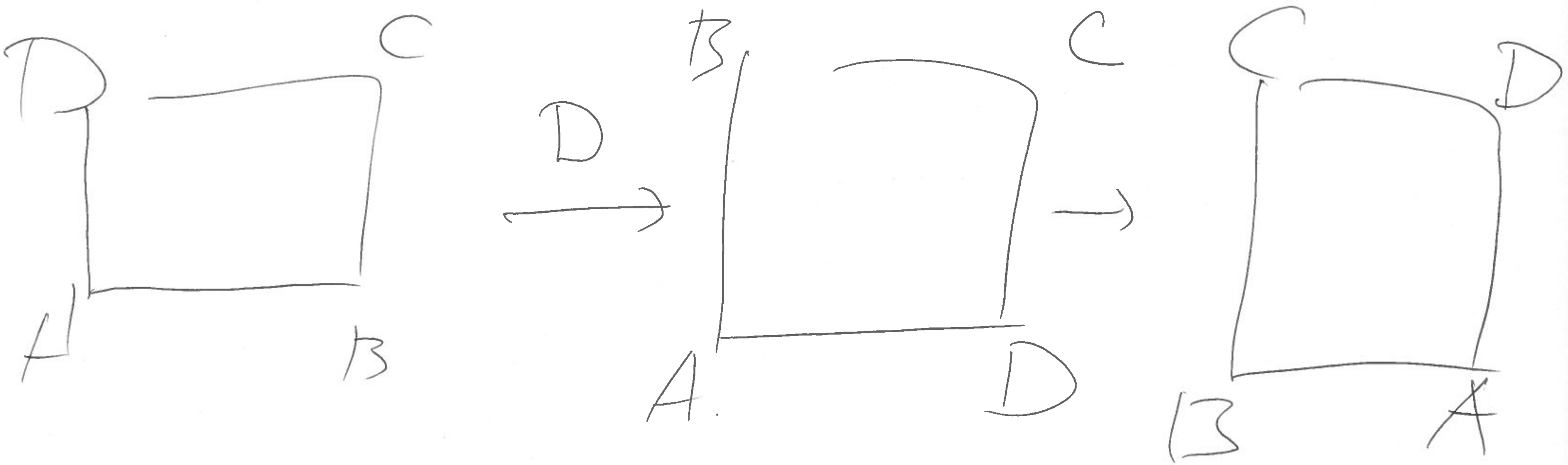
$\cancel{R_{450}}, H, V, D, D'$

$R_{90} \circ D$

$R_{90} \circ D = V$ $\qquad$ $D \circ R_{90} = H$

H $\qquad$ f

New



$\cancel{(f \circ g)(x) = f(g(x))}$ $\qquad$ V

	R_0	R_{90}	R_{180}	R_{270}	D	D'	H	V
R_0	R_0	R_{90}	R_{180}					
R_{90}					V			
R_{180}								
R_{270}								
D		H						
D'								
H								
V								

D_4 is closed under composition

(i) $R_0 \circ S = S$ $\quad R_0$ unity

$S \circ R_0 = S$

(ii) $\forall S \in D_4 \ \exists T \in D_4$ s.t. $T \circ S = S \circ T = R_0$

we write $T = S^{-1}$.

(iii) $\forall S, T, U \in D_4 \quad S \circ (T \circ U) = (S \circ T) U$

associativity

$\Rightarrow (D_4, \circ)$ is a group

$\mathcal{G}$ a ~~set~~ $\circ : \mathcal{G} \times \mathcal{G} \to \mathcal{G}$

(i) $\exists e \in \mathcal{G}$ s.t. $\forall g \in \mathcal{G}$ $e \cdot g = g \cdot e = g$
unit element

(ii) $\forall g \in \mathcal{G}$ $\exists h \in \mathcal{G}$ s.t. $g \circ h = e = h \circ g$.

(iii) $\forall g, h, j \in \mathcal{G}$: $(g \circ h) \circ j = g \circ (h \circ j)$

$(\mathcal{G}, \circ)$ forms a group

if $\forall g, h \in \mathcal{G}$: $g \circ h = h \circ g$ then we say that $(\mathcal{G}, \circ)$ is commutative or Abelean

Examples: $(\mathbb{Z},+)$ ✓, $(\mathbb{N},+)$ ✗, $(\mathbb{Q},\cdot)$ ✗

$(\mathbb{Q}\setminus\{0\},\cdot)$, $(\mathbb{R}\setminus\mathbb{Q},\cdot)$ ✗;

in D_4. $R_{90+k\cdot 360} = R_{90}$

$\mathbb{Z}_5 = \{[0],[1],[2],[3],[4]\}$. is a group, $\mathbb{Z}_n$

$[1]+[3] = [a+b]$

$= 6+8 = 14$

$11 - 2 = 9$

$[1]+[3]=[4]$

b

Unicity of e?

suppose that we have two unit elements:

e_1 and $e_2 \in G$: $\forall g \in G$:

$$e_j \cdot g = g \cdot e_j \quad j = 1, 2$$

$$e_1 \underset{\substack{\uparrow \\ \text{because} \\ e_2 \text{ is unity}}}{=} e_1 \cdot e_2 \underset{\substack{\uparrow \\ \text{because} \\ e_1 \text{ is unity.}}}{=} e_2$$

suppose $a \circ b = a \circ c \Rightarrow b = c$ if $a \neq e$

$a = e$ $\quad \underbrace{e \cdot b}_{b} = \underbrace{e \cdot c}_{c} \Rightarrow b = c.$

$$\frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

Caution when cancelling common symbols

$g \in G$

$e = g \circ h_1 = g \cdot h_2 \Rightarrow h_1 = h_2.$