

## ADS practice questions: Binary trees

### Question 1.

Prove the following proposition:

Depth  $d$  of a binary tree has a maximum of  $2^d$  nodes.

### Question 2.

Prove the following proposition:

A binary tree with height  $h$  has a maximum of  $2^{h+1} - 1$  nodes.

### Question 3.

Prove the following proposition:

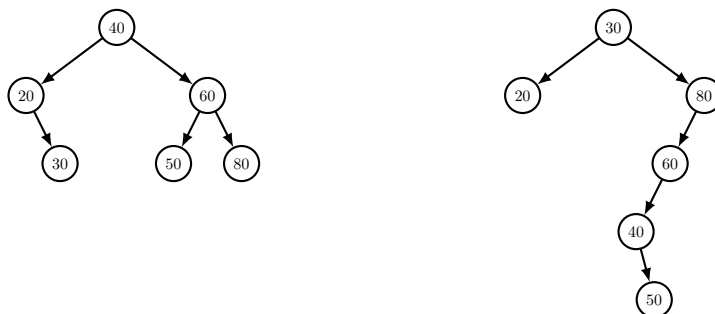
A binary tree with  $n$  nodes has a height of at least  $\lceil \log(n + 1) \rceil - 1$ .

### Question 4.

Given a binary tree with positive integer keys. Give an algorithm that determines whether the tree is sorted in order.

### Question 5.

Take the following BSTs:



1. In both trees, insert first 70 and then 40.
2. From the above results, delete first 60 and then 70.

### Question 6.

Give the definition of a *balanced* binary tree. Then give a criterion for the balancedness of a binary tree of  $n$  nodes in terms of the depth of the tree.

### Question 7.

Take the second BST in question 5, and transform it into a balanced tree using rotations.

### Question 8.

1. Insert, in sequence, the elements 10, 20, ..., 70 in an initially empty BST. How balanced is the result?
2. Now repeat the above, but in this case rotate the tree to restore its balance when the insertion of an element unbalances the tree.