Statistical Techniques for CS/BIT

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General Information & Planning

Course Materials

Assignments

SPSS

Sample Tests

Today:

Lectorial (meeting #8)

- Lecture CH 4 (4.2)

  10:45-11:30
- Break

  11:30-11:45
- Tutorial

  11:45-12:30

(online: join the Teams channel of Katharina for questions)

Test for the population mean μ, σ^2 unknown
Test for the population mean μ, σ unknown

A search engine is designed to carry out web searches, to supply the search results with a response time of 0.50 seconds. Researchers want to investigate whether this search engine always reaches this response time.

Researchers test hundred times for the response time. They arrive a mean response time of 0.64 seconds with a variance of 0.38. It follows from the data that the assumption of a normal distribution is realistic.

Does the search engine achieve the expected response time?

Do we conclude that this search engine does not carry out the web searches within the expected response time?

Perform the test with significance level 0.01.

Test for the population mean μ, σ^2 unknown

$\overline{x}$=0.64 s, s^2= 0.38, n=100

1. Model: the response times $X_1, \ldots, X_{100} \sim$ N(μ, σ^2) i.i.d., μ and σ^2 unknown
2. H_0: μ=0.50, H_1: μ>0.50 , $\alpha = 0.01$
3. Test Statistic: $T = \frac{\overline{X} - 0.50}{S/\sqrt{100}}$

- T measures to what extend $\bar{X}$ deviates from 0.50, normalized by $S/\sqrt{\mathrm{n}}$
- If H_0 is true, then T has a known distribution: $\mathrm{t}_{\mathrm{n}-1}$

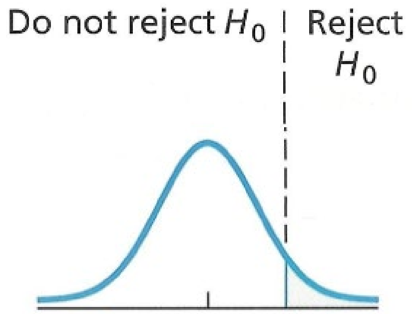
Test for the population mean μ, σ^2 unknown

$\bar{x}$=0.64 s, s^2= 0.38, n=100

1. Model: the response times $X_1, \ldots, X_{100}$ ~ N(μ, σ^2) i.i.d., μ and σ^2 unknown
2. H_0: μ=0.50, H_1: μ>0.50 , $\alpha = 0.01$
3. Test Statistic: $T = \dfrac{\bar{X} - 0.50}{S/\sqrt{100}}$
4. If H_0 is true then T has a t_{99} distribution: $T \sim t_{99}$
5. Observed value: $t = \dfrac{0.64 - 0.50}{0.62/\sqrt{100}} \approx 2.258$
6. Right-sided test, P($T_{99} \geq c$) = 0.01

C = 2.364

Rejection Region/Critical Region: "Reject H_0, if $T \geq c$"

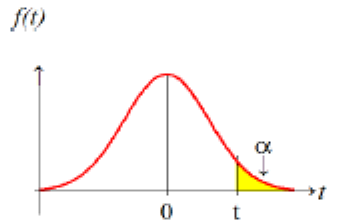


Tab-2

Table t-distribution

In the table you find the critical values t for the upper-tailed probabilities such that

$$P(T \geq t) = \alpha$$



Number of degrees of freedom	α							
	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	1.000	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.679	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
70	0.678	1.294	1.667	1.994	2.381	2.648	3.211	3.435
80	0.678	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.677	1.290	1.660	1.984	2.364	2.626	3.174	3.390
120	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

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2. H_0: μ=0.50, H_1: μ>0.50 , $\alpha = 0.01$
3. Test Statistic: $T = \dfrac{\overline{X} - 0.50}{S/\sqrt{100}}$
4. If H_0 is true then T has a t_{99} distribution: $T \sim t_{99}$
5. Observed value: $t = \frac{0.64 - 0.50}{0.62/\sqrt{100}}$ ≈ 2.258
6. Right-sided test, P($T_{99} \geq c$) = 0.01, c= 2.364

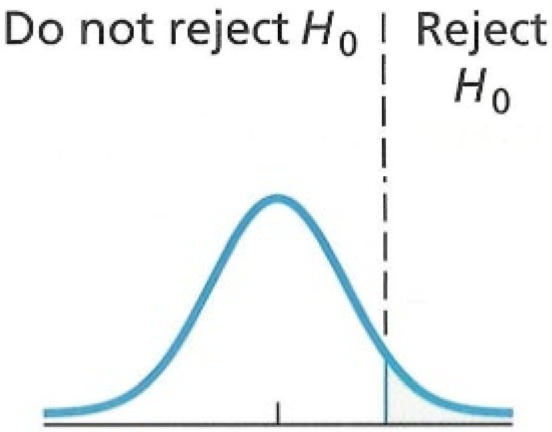
Rejection Region/Critical Region: "Reject H_0, $if\ T \geq c$"

7. Statistical decision: t = 2.258 < 2.364, fail to reject H_0
8. There is insufficient statistical evidence that this search engine does not carry out the web searches within the expected response time.

Right-sided Test

H_0: $\mu = \mu_0$,
H_1: $\mu > \mu_0$

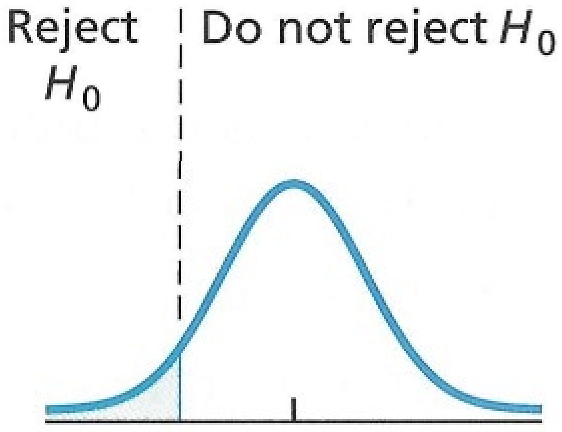
$\mathcal{P}(T_{n-1} \geq c) = \alpha$



Left-sided Test

H_0: $\mu = \mu_0$,
H_1: $\mu < \mu_0$

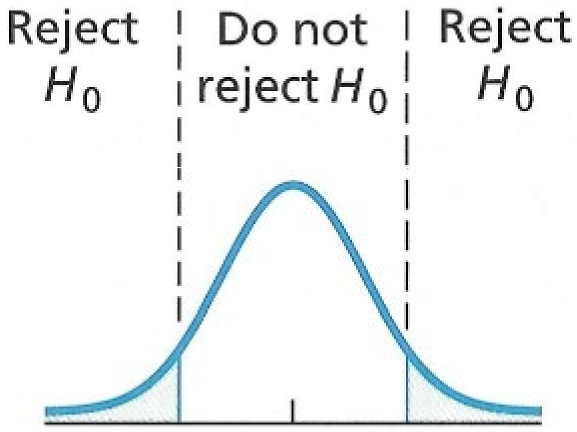
$\mathcal{P}(T_{n-1} \leq c) = \mathcal{P}(T_{n-1} \geq -c) = \alpha$



Two-sided Test

H_0: $\mu = \mu_0$,
H_1: $\mu \neq \mu_0$

$\mathcal{P}(T_{n-1} \geq c) = \frac{\alpha}{2}$



Questions ?

Thank you!