Artificial Intelligence
Tutorial 6 on Neural Networks
Including Answers

Introduction

The following questions about Neural Networks are examples of typical questions one can expect on the AI exam, but the exam questions are MC. After the tutorial the answers to the MC will be available on BB.

Questions on Neural Networks

1. Assume that we are training a logistic classifier (linear classifier with logistic regression) using the `Logistic` function (called the `sigmoid` function in the slides) and that the current logistic classifier has the weights $(w_0, w_1, w_2) = (1, 2, -1)$. The next feature point in our training set is given by $x = (1, -1)$.

   (a) What is the output (activation) of this logistic classifier for the input x?

   (b) How will the feature point x be classified, 0 or 1, given the current weights $w = (1, 2, -1)$ of the linear classifier?

   (c) Assume that the feature point x is misclassified and we use the L_2 (quadratic) loss function $(y - h_{\mathbf{w}}(x))^2$. How will the weights of the linear classifier be adapted. Assume a learning rate α of 0.7.

   (d) How will x be classified after the above adaptation of the weight vectors w? Is this adaptation a step in the right direction? **Motivate your answer!**

   Answers: For the answers we use the terminology and notation of the course book.

   (a) The output is $h_{\mathbf{w}}(x) = \texttt{Logistic}(\mathbf{w}.\mathbf{x}) = \texttt{Logistic}(4) = 0.9820$.

   (b) Since $h_{\mathbf{w}}(x) > 0.5$ x will be classified as 1.

   (c) Since x misclassified the target value $y = 0$. The adaption of the weights is given by formula (18.8) in the course book. First we compute

   $$\alpha \times (y - h_{\mathbf{w}}(x)) \times h_{\mathbf{w}}(x) \times (1 - h_{\mathbf{w}}(x)),$$

   which equals $0.7 \times (0 - 0.982) \times 0.982 \times (1 - 0.982) = -0.0121$.
   Remark: Observe that the factor 2 of the derivative of the quadratic loss

function L_2 is incorporated in the learning parameter α and not taken into account. See text after formula 18.5. Another option, which is also used often, would be to change the quadratic loss function to $\frac{1}{2}(y - h_{\mathbf{w}}(x))^2$.
From this we can easily calculate the new weights, see 18.8.

$$w_i \leftarrow w_i + -0.0121 \times x_i.$$

Now the dummy input $x_0 = 1$. Hence the new value for w_0 is 0.9879. In a similar fashion the new w_1 is 1.9879 and the new w_2 is -0.9879.

(d) For these new values of w the output of the classifier is 0.9814. It is decrease in comparison to the previous value and a step towards the target output 0 and hence a (small) step in the right direction.

2. Consider the Neural Network of Figure 18.22 of the book, also depicted below. As stated in the text, the dummy inputs are not shown!

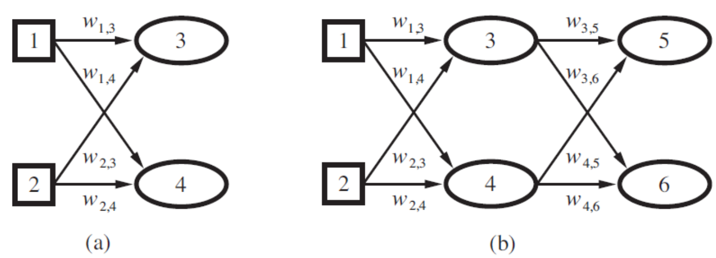

Figure 18.20 **FILES: figures/neural-net.eps (Wed Nov 4 11:08:22 2009).** (a) A perceptron network with two inputs and two output units. (b) A neural network with two inputs, one hidden layer of two units, and one output unit. Not shown are the dummy inputs and their associated weights.

More specific we consider the NN on the left and assume the following weights: $w_{0,3} = 2$, $w_{1,3} = 1$, $w_{2,3} = -2$, $w_{0,4} = 1$, $w_{1,4} = 2$, $w_{2,4} = -1$. The activation function of the neurons 3 and 4 is the `Logistic` function.

(a) What is the output of this NN on the input $x = (1, 2)$?

(b) The target output for x is $(0, 1)$. What are the delta's $\Delta(3)$ and $\Delta(4)$ for the neurons 3 and 4 if we assume the L_2 loss function?

(c) How will the weights of NN be adapted if the learning parameter for all neurons is $\alpha = 0.5$?

Answers: For the answers we use the terminology and notation of the course book.

(a) The computation of the output for each neuron is similar to the previous exercise. Output for neuron 3 is `Logistic`$(-1) = 0.2689$. Output of neuron 4 is `Logistic`$(1) = 0.7311$.

(b) The target values are $(0, 1)$ and we can compute the Δ's
Remark: The delta's are based on Back-Propagation Algorithm 8.24 in the book. Observe that the factor 2 of the derivative of the quadratic loss function L_2 is incorporated in the learning parameter α and not taken into account. See text after formula 18.5. Another option, which is also used often, would be to change the quadratic loss function to $\frac{1}{2}(y - h_{\mathbf{w}}(x))^2$.
Computing $\Delta(3) = (0 - 0.2689) \times 0.2689 \times (1 - 0.2689) = -0.0529$. And $\Delta(4) = (1 - 0.7311) \times 0.7311 \times (1 - 0.7311) = 0.0529$. Here we use that the derivative of the `Logistic` function g is given $g'(z) = g(z)(1 - g(z))$.

(c) In order to compute the weight updates we will use the formula (18.11)

$$w_{j,k} \leftarrow w_{j,k} + \alpha \times a_j \times \Delta_k$$

Now we can compute the new weights. $w_{3,0} = 2 + 0.5 \times 1 \times \Delta(3) = 1.9471$ and $w_{3,1} = 1 + 0.5 \times 1 \times \Delta(3) = 0.9471$ and $w_{3,2} = -2 + 0.5 \times 2 \times \Delta(3) = -2.1058$. The new weights for neuron 4 have the value $w_{0,4} = 1.0264$, $w_{1,4} = 2.0264$ and $w_{2,4} = -0.9471$.

3. Consider the Neural Network of Figure 18.22 of the book, also depicted below. As stated in the text, the dummy inputs are not shown!

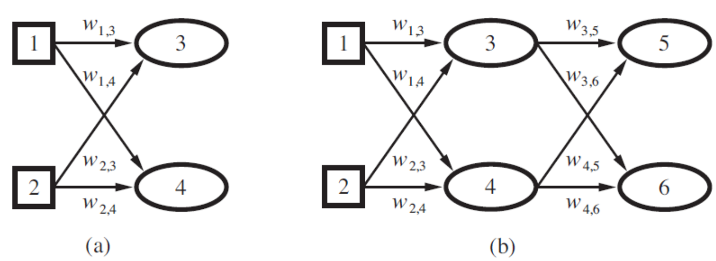

Figure 18.20 **FILES: figures/neural-net.eps (Wed Nov 4 11:08:22 2009).** (a) A perceptron network with two inputs and two output units. (b) A neural network with two inputs, one hidden layer of two units, and one output unit. Not shown are the dummy inputs and their associated weights.

More specific we consider the NN on the right and assume the following weights: $w_{0,3} = 2$, $w_{1,3} = 1$, $w_{2,3} = -2$, $w_{0,4} = 1$, $w_{1,4} = 2$, $w_{2,4} = -1$, $w_{0,5} = 0$, $w_{3,5} = -1$, $w_{4,5} = -1$ and $w_{0,6} = -1$, $w_{3,6} = 1$, $w_{4,6} = -2$. The activation function of the hidden neurons 3 and 4 is the `Logistic` function and the activation function of the output neurons 5 and 6 is the identity function $g(x) = x$.

(a) What is the output of this NN on the input $x = (1, 2)$?

(b) The target output for x is $(0, 1)$. What are the delta's $\Delta(5)$ and $\Delta(6)$ for the output neurons if we assume the L_2 loss function?

(c) What are the delta's $\Delta(3)$ and $\Delta(4)$ for the hidden neurons?

(d) How will the weights of NN be adapted if the learning parameter for all neurons is $\alpha = 0.5$?

Answers: For the answers we use the terminology and notation of the course book.

(a) We already computed the output of neuron 3 and 4 in the previous exercise. The activation function of the output neurons 5 and 6 is the identity function, hence the output of neuron 5 is given by $w_{0,5} + w_{3,5} \times a_3 + w_{4,5} \times a_4 = 0 + -1 \times 0.2689 + -1 \times 0.7311 = -1.0000$. And the output of neuron 6 is $-1 + 1 \times a_3 + -2 \times a_4 = -2.1932$.

(b) Target values are $(0, 1)$ and the derivative of the identity function is 1. So $\Delta(5) = (0 - -1.0000) = 1.000$ and $\Delta(6) = (1 - -2.1932) = 3.1932$.

(c) The delta's for the hidden neuron can be computed by back-propagation. Using that the derivative of the `Logistic` function g is given $g'(z) = g(z)(1 - g(z))$.

$$\begin{aligned}\Delta(4) &= a_4 \times (1 - a_4) \times [w_{4,5} \times \Delta(5) + w_{4,6} \times \Delta(6)] \\ &= 0.7311 \times (1 - 0.7311) \times [-1 \times 1.0000 + -2 \times 3.1932] \\ &= -1.4522\end{aligned}$$

In a similar fashion

$$\begin{aligned}\Delta(3) &= a_3 \times (1 - a_3) \times [w_{3,5} \times \Delta(5) + w_{3,6} \times \Delta(6)] \\ &= 0.2689 \times (1 - 0.2689) \times [-1 \times 1 + 1 \times 3.1932] \\ &= 0.4312\end{aligned}$$

(d) Now we have all the delta's and can update the weights once again using formula (18.11). This results in the following new weights: $w_{0,3} = 2.2156$, $w_{1,3} = 1.2156$, $w_{2,3} = -1.5688$, $w_{0,4} = 0.2739$, $w_{1,4} = 1.2739$, $w_{2,4} = -2.4522$, $w_{0,5} = 0.5000$, $w_{3,5} = -0.8655$, $w_{4,5} = -0.6345$ and $w_{0,6} = 0.5966$, $w_{3,6} = 1.4294$, $w_{4,6} = -0.8328$.
One should check the with new weights the outputs are closer to the desired outputs for $x = (1, 2)$.