Artificial Intelligence
Exercises for Tutorial 3 on Probabilistic Inference and Bayesian Networks

Introduction

The following multiple choice questions are examples of typical questions one can expect on the AI exam. The questions on the AI exam are also multiple choice, but for this tutorial one has to explain the answers given. Moreover at the end one can find some open questions.

Exercises

1. Consider the following case of a car accident that involved a taxi.

   All taxis in town are blue or green. It is known that under dim lighting conditions discrimination between blue and green is 70% reliable; which means that $P(W = b|C = b)$ as well as $P(W = g|C = g)$ are 0.70, where C is the two valued variable with values g and b indicating the color of the taxi, $C = b$ means *"the taxi is blue"* and W is the two valued variable indicating the declaration of the witness, $W = b$ means *"the witness says the taxi is blue"*.

   Suppose that 7 out of 10 taxis are actually green.

   a: A witness declares that the taxi was blue. Given the declaration of our witness what is the probability that the taxi is indeed blue?

   b: Suppose two witnesses independently declare that the taxi was blue. Draw the Bayesian Network for this case and what is the probability that the taxi was indeed blue?

   c: Now assume that a third *independent* witness appears on the scene and declares that the taxi was green. What is the now probability that the taxi was indeed blue?

2. The Prosecution argument. The counsel for the prosecution argues as follows:

   > *Ladies and gentlemen of the jury, the probability of the observed match between the sample at the scene of the crime and that of the suspect having*

arisen by innocent means is 1 in 10 million. This is an entirely negligible probability, and we must therefore conclude that with a probability overwhelmingly close to 1 that the suspect is guilty. You have no alternative but to convict.

This argument is known as the Prosecutor's Fallacy. Explain the error in the counsel's reasoning.

3. "Most car accidents are caused by people that do have a driver's licence." What is suggested by this statement? What are the relevant conditional probabilities?

4. Make exercise 14.1 from the book of Russel and Norvig Artificial Intelligence (3rd edition).

5. Make exercise 14.4 from the book of Russel and Norvig Artificial Intelligence (3rd edition).

6. Given the Sprinkler network shown in Figure 1. What is the best approximation of the value of $P(S = True|W = True)$ (the probability that the **S**prinkler was on given that the grass is **W**et)? Use the enumeration method. Indicate where you use the "conditional independency" relation represented by the BN.

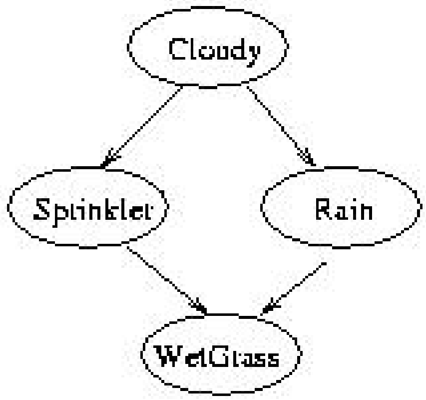
   (a) 0.2781
   (b) 0.6471
   (c) 0.1945
   (d) 0.4298

7. Given the Sprinkler network shown in Figure 1. What is the best approximation of the value of $P(S = True|W = True, R = True)$ (the probability that the **S**prinkler was on given that the grass is **W**et and that it was **R**aining)?

   (a) 0.2781
   (b) 0.6471
   (c) 0.1945
   (d) 0.4298

8. In the Bayesian Network below with three boolean variables the probabilities for P and M are: $P(M = true) = 0,1$ and $P(L = true) = 0.7$ and the conditional probabilities for variable V are as shown in the table.

P(C=F)	P(C=T)
0.5	0.5

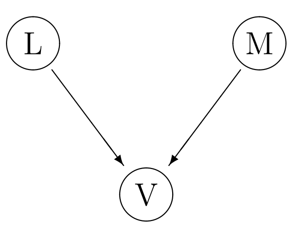


C	P(S=F)	P(S=T)
F	0.5	0.5
T	0.9	0.1

C	P(R=F)	P(R=T)
F	0.8	0.2
T	0.2	0.8

S	R	P(W=F)	P(W=T)
F	F	1.0	0.0
T	F	0.1	0.9
F	T	0.1	0.9
T	T	0.01	0.99

Figure 1: The Sprinkler Bayesian network



L	M	$P(V = true \mid L, M)$
true	true	0,9
true	false	0,5
false	true	0,3
false	false	0,05

What is the value of $P(V = true \mid L = true)$?

(a) 0.72

(b) 0.54

(c) 0.46

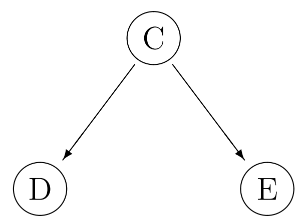


Figure 2: A Bayesian Network.

(d) 0.28

9. Consider the Bayesian Network in Figure 2.

   All three nodes represent boolean variables. The probability distributions for the nodes of the network are as follows.

   For node C: $P(C = true) = 0.4$.

   For node D: $P(D = true|C = true) = 0.8$ and $P(D = true|C = false) = 0.3$.

   For node E: $P(E = true|C = true) = 0.9$ and $P(E = true|C = false) = 0.2$.

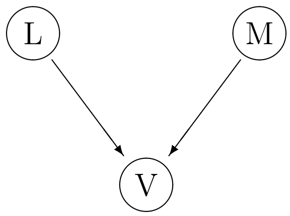
   What is the value of $P(D = true)$?

   (a) 0.50
   (b) 0.32
   (c) 0.18
   (d) 0.90

10. Consider again the Bayesian Network in Figure 2 with the probability distributions as given in the exercise above. One of the following statements is true. Which one?

    a) $P(D = true|E = true) > P(D = true)$
    b) $P(D = true|E = true) = P(D = true)$
    c) $P(D = true|E = true) < P(D = true)$
    d) There is not enough information to compute $P(D = true|E = true)$.

11. In the Bayesian Network below with three boolean variables the probabilities for P and M are: $P(M = true) = 0,2$ and $P(L = true) = 0.7$ and the conditional probabilities for variable V are as shown in the table.



L	M	$P(V = true \mid L, M)$
true	true	0,9
true	false	0,5
false	true	0,3
false	false	0,05

What is the value of $P(V = false \mid L = false)$?

(a) 0.3

(b) 0.7

(c) 0.9

(d) 0.1

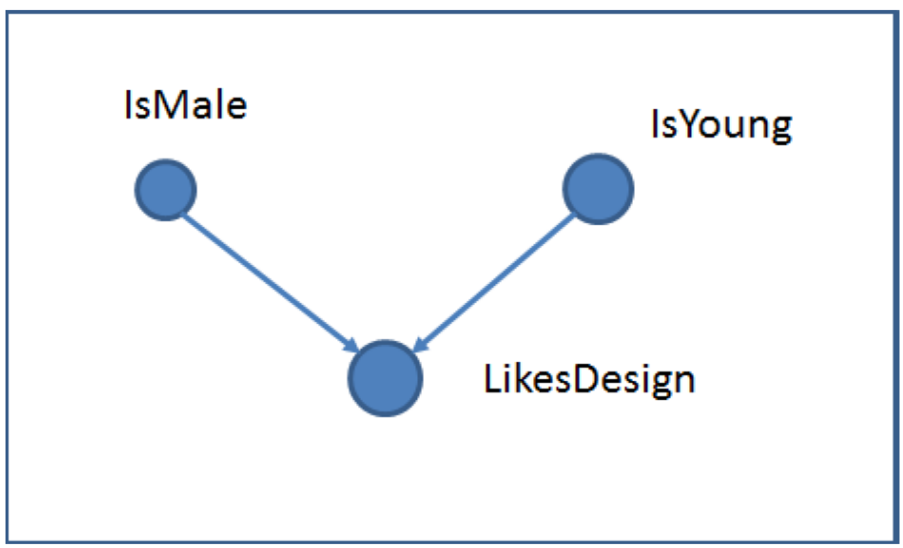


Figure 3: The dependencies for the user tests of my design

12. The Bayesian Network structure given in Figure 3 models the dependencies between three properties related to my design. IsMale is true when the user is male, IsYoung is true when the user is young, LikesDesign is true when the user likes my design. Which of the following statements is true? Give a proof or counter example.

    a) IsYoung and IsMale are independent

    b) IsYoung and IsMale are independent given LikesDesign