Artificial Intelligence
Exercises for Tutorial 2 on Predicate Logic

November 14, 2021

Introduction

The following multiple choice questions are examples of typical questions one can expect on the AI exam. The questions on the AI exam are also multiple choice, but for this tutorial one has to explain the answers given. Moreover at the end one can find some open questions. After the tutorial the answers to the MC will be available on Canvas. FOPL stands for First Order Predicate Logic.

Exercises

1. Aristotle studied reasoning with noun phrases like “every man”, “some man” and “no man”. The four sentence forms Aristotle treated in his syllogisms theory are:

   (a) All P’s are Q’s (e.g., all men are mortal)

   (b) Some P’s are Q’s (e.g., some men are mortal)

   (c) No P’s are Q’s (e.g., no men is mortal)

   (d) Some P’s are not Q’s (e.g., some men are not mortal)

   Translate these four sentences into well-formed formulas of FOPL.

2. Translate the following sentences into FOPL:

   (a) “A small, happy dog is at home”

   (b) “Every small dog that is at home is happy”

3. Given are the following predicates:

   - $P(x)$

- $Q(x)$

Now consider the following sentence: $\neg\forall x\ (P(x) \wedge Q(x))$. Which of the following models satisfies this sentence?

(a) No objects and relations P, Q.

(b) One object a and relations P, Q.

(c) One object a and relations P, Q with $a \in P$ and $a \in Q$.

4. Given are the following three predicates:

   - $Food(x)$: x is something to eat.
   - $Person(x)$: x is a person.
   - $Likes(x, y)$: x likes y

   Now consider the following two sentences in first-order logic:

   (I) $\forall x\ (Food(x) \Rightarrow \exists y\ (Person(y) \wedge Likes(y, x)))$
   (II) $\exists x\ (Food(x) \wedge \forall y\ (Person(y) \Rightarrow Likes(y, x)))$

   Which of the following statements about these two sentences is correct? Construct simple models, i.e., a set of objects and a relation for each predicate (i.e., a relation symbol and a specification of which objects are part of that relation), over which the truth of the sentences can be evaluated, to find an answer.

   (a) (I) and (II) are equivalent.

   (b) (I) and (II) are not equivalent, but (I) follows from (II).

   (c) (I) and (II) are not equivalent, but (II) follows from (I).

   (d) (I) and (II) are not equivalent, and neither follows from the other.

5. **Not for Create**
   Consider the two statement in the language of first-order logic:

   I.) $\forall x(P(x) \Rightarrow \exists y Q(y))$
   II.) $\forall x \neg P(x) \vee \neg\forall y \neg Q(y)$

   The question is whether there is an entailment relation between these two sentences. We write $P \models R$ for "P entails R" and $P \not\models R$ for "P does not entail R".Only one of the following is true; which one?

   (a) I $\models$ II but II $\not\models$ I.

   (b) I $\not\models$ II but II $\models$ I.

   (c) I $\not\models$ II and II $\not\models$ I.

(d) I $\models$ II and II $\models$ I.

Remember that, for any two sentences in first order logic φ and ψ, $\varphi \vDash \psi$ holds if $\varphi \vdash \psi$. You can check whether $\varphi \vdash \psi$ by means of resolution: find out whether $\varphi \wedge \neg\psi$ is a contradiction. For checking whether I $\models$ II and II $\models$ I you need to apply the resolution procedure two times.

Alternatively, you may try to prove directly by rewriting the formulas using logical equivalences.

6. **Not for Create**
   Consider the formula $Knows(John, x)$. We try to unify this formula with different other formula. In one of these cases unification fails. For which one unification will fail?

   (a) $Knows(John, Jane)$

   (b) $Knows(y, Bill)$

   (c) $Knows(y, Mother(y))$

   (d) $Knows(x, Jane)$

7. **Not for Create**
   We want to Skolemise the following sentence in first-order logic:

   $$\forall x \forall y \exists z \forall v \ (P(x, y) \wedge Q(v) \Rightarrow R(x, y, z, v))$$

   Of the following four substitutions, only one produces a correct Skolemisation. Which one?

   (a) $\{z/S\}$

   (b) $\{z/S(x)\}$

   (c) $\{z/S(v)\}$

   (d) $\{z/S(x, y)\}$

   The idea is that you correctly choose whether to supply a Skolem constant or a Skolem function. If the answer is a Skolem function, you further have to choose of which variables it is a function.

8. **Not for Create**
   Let $Child(x, y, z)$ mean x is child of y and z.
   (a) Express in FOPL that a child of z and y is a child of y and z.
   (b) Let a knowledge base KB contain this formula as well as the fact $Child(Eve, Anne, Oscar)$.
   Prove using natural deduction (=applying inference rules) that $Child(Eve, Oscar, Anne)$,
   in particular by applying the rules of Modus Ponens and elimination of quantifiers.

9. **Not for Create**

A knowledge base (KB) contains the two statements:

$$\forall x(P(x, b) \vee Q(x)) \qquad (1)$$

$$\forall y(\neg P(f(y), b) \vee Q(y)) \qquad (2)$$

Show (taking the steps of a proof by resolution) that the following is derivable from this KB.

$$\forall y(Q(y) \vee Q(f(y))) \qquad (S)$$