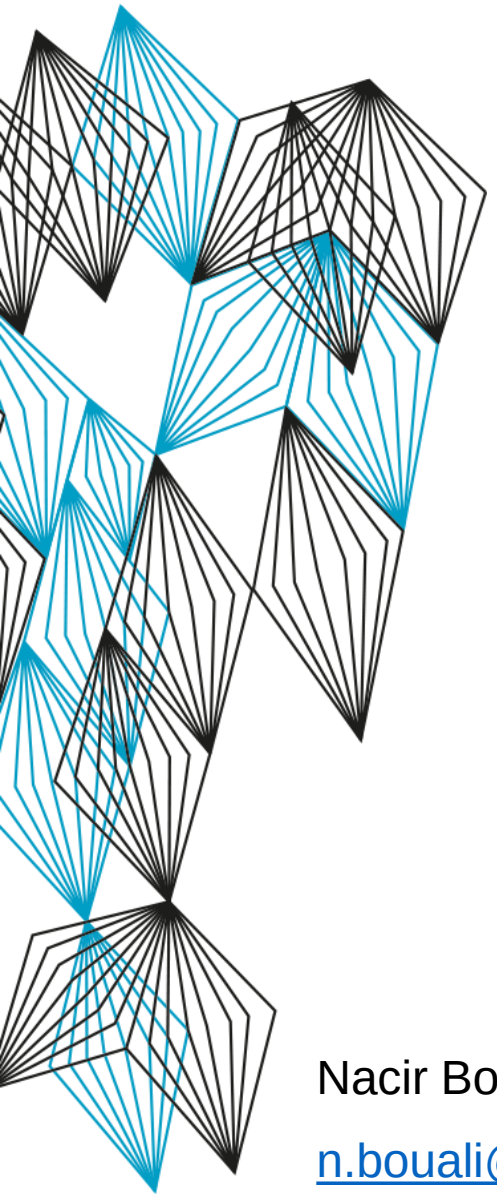

ARTIFICIAL INTELLIGENCE & CYBER SECURITY

PROBABILISTIC REASONING
(CONDTIONAL) INDEPENDANCE

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INDEPENDENCE

- Random variables A and B are said to be independent if
  - P(A|B)=P(A)
  - P(B|A)=P(B)
  - P(A,B)= P(A) P(B)

Covid	Toothache	Probability
Yes	Yes	0.04
Yes	No	0.36
No	Yes	0.06
No	No	0.54

  - P(covid)=0.4
  - P(toothache)=0.1
  - If you check for P(Covid, Toothache), you will find that indeed these random variables are independent.

INDEPENDENCE

- Independence is quite powerful, but very rare.
- Makes inference more efficient as the probability distributions we need to store are minimized.
- Only 2 values to store instead of 4.

Covid	Toothache	Probability
Yes	Yes	0.04
Yes	No	0.36
No	Yes	0.06
No	No	0.54

CONDITIONAL INDEPENDENCE

- Assume that we have a coin, and a reason to believe that the coin is biased towards heads, but we’re not sure.
- Let Bob and Alice be two people to toss the coin, and let A represent the variable of Alice’s toss, and B repesent the variable of Bob’s toss.
- A and B in this case are not independent.
- If Alice tosses first and lands on heads, we will increase our belief that Bob’s toss will land on head too.
- P(B=heads|A=heads)>P(A=heads)
- Add a new boolean random variable C (the coin is biased towards heads). Though A and B are not independent, once we know for sure the value of C then an evidence from A cannot change our belief about B.
- P(A|C)=P(A|B,C)

CONDITIONAL INDEPENDENCE

- $P(A|C)=P(A|B,C)$
- We can say that A and B are conditionally independent on C.
- With the chain rule we can rewrite $P(A,B,C)$ as $P(A|B,C)P(B,C)$ $=P(A|B,C)P(B|C)P(C) = P(A|C)P(B|C)P(C)$

➢ We can thus store 2+2+1 = 5 parameters in our table instead of 8 (7).

➢ Allows to reduce the complexity of representing joint distributions from $O(2^n)$ to a linear O(n).

➢ This forms the foundation of a Bayesian Network BN.