



# ARTIFICIAL INTELLIGENCE & CYBER SECURITY

## PROBABILISTIC REASONING RULES

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# CHAIN RULE

- Product rule states the following;
  - $P(\text{headache}, \text{covid}) = P(\text{headache}|\text{covid}) * p(\text{covid}) = P(\text{covid}|\text{headache}) * p(\text{headache})$
  - $P(A,B)=P(A|B)*P(B)=P(B|A)*P(A)$

Headache	Fever	Covid	Probability
Yes	Yes	Yes	0.25
Yes	Yes	No	0.12
Yes	No	Yes	0.05
Yes	No	No	0.05
No	Yes	Yes	0.10
No	Yes	No	0.10
No	No	Yes	0.01
No	No	No	0.32



# CHAIN RULE

- For n variables  $X_1, X_2, \dots, X_n$

- $$\begin{aligned} P(X_1, \dots, X_n) &= P(X_1, \dots, X_{n-1})P(X_n | X_1, \dots, X_{n-1}) \\ &= P(X_1, \dots, X_{n-2})P(X_{n-1} | X_1, \dots, X_{n-2})P(X_n | X_1, \dots, X_{n-1}) = \dots \\ &= \prod_{i=1}^n P(X_i | X_1, \dots, X_{i-1}) \end{aligned}$$



# CHAIN RULE

$$2^3 \leftrightarrow 2^{10^0} \approx 10^3$$

- The chain rule is very powerful, and holds true regardless of the distribution  $X_1..X_n$
- The chain rule implicitly says that instead of specifying all joint probabilities  $P(X_1, \dots, X_n)$ , you can specify only  $P(X_1), P(X_2|X_1), P(X_3|X_1, X_2), \dots, P(X_n|X_1, \dots, X_{n-1})$
- We could have for example specified the following probabilities;
  - $P(\text{covid})=0.41, P(\neg\text{covid})=0.59$
  - $P(\neg\text{headache}|\neg\text{covid})=0.71, P(\text{headache}|\neg\text{covid})=0.29$
  - $P(\neg\text{headache}|\text{covid})= 0.27 , P(\text{headache}|\text{covid})= 0.73$
- Enough ingredients to calculate  $P(\text{Headache}, \text{Covid})$ 
  - $P(\text{headache}, \text{covid})=P(\text{headache}|\text{covid})P(\text{covid}) = 0.73*0.41=0.30$
  - $P(\neg\text{headache}, \text{covid})=P(\neg\text{headache}|\text{covid})P(\text{covid})=0.27*0.41=0.11$
  - $P(\text{headache}, \neg\text{covid}) = P(\text{headache}|\neg\text{covid})P(\neg\text{covid})=0.29*$
- What if I want to calculate  $P(\neg\text{covid}|\text{headache})$ ?

Headache	Fever	Covid	Probability
Yes	Yes	Yes	0.25
Yes	Yes	No	0.12
Yes	No	Yes	0.05
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No	Yes	Yes	0.10
No	Yes	No	0.10
No	No	Yes	0.01
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# BAYES RULE

- What if I want to calculate  $P(\neg covid|headache)$ ?
- $P(\neg covid|headache)$
- $= \frac{P(\neg covid, headache)}{p(headache)} = \frac{P(headache|\neg covid)p(\neg covid)}{p(headache)}$
- $= \frac{P(headache|\neg covid)p(\neg covid)}{p(headache, covid) + P(headache, \neg covid)}$
- $= 0.29 * 0.59 / (0.30 + 0.17) = 0.36$

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No	No	No	0.32

$$\frac{0.17}{0.47}$$



# BAYES RULE

$$P(A, B) = P(A|B)P(B) \\ = P(B|A)P(A)$$

- Bayes rule gives the below general formula

$$P(\underline{Cause}|\underline{Effect}) = \frac{P(\underline{Effect}|\underline{Cause})P(\underline{Cause})}{P(\underline{Effect})}$$

Posterior

Likelihood

Prior

$$P(A|B) \\ = \frac{P(B|A)P(A)}{P(B)}$$

