# Discrete Mathematics for Computer Science, 26 October 2018; Part 2
## Solution/Correction standard

4. Basis step for $n = 0$, $n = 1$ and $n = 2$:

$$a_0 = 1 \leq \left(\sqrt{2}\right)^0; \quad a_1 = 1 \leq \left(\sqrt{2}\right)^1; \quad a_2 = 2 \leq \left(\sqrt{2}\right)^2. \qquad \textbf{[1 pt]}$$

Induction step: Let $k \geq 2$ and suppose that for all $0 \leq p \leq k$:

$$a_p \leq \left(\sqrt{2}\right)^p. \qquad \text{(Induction Hypothesis)} \qquad \textbf{[2 pt]}$$

We must show that the induction hypothesis implies:

$$a_{k+1} \leq \left(\sqrt{2}\right)^{k+1}. \qquad \textbf{[1 pt]}$$

Well,

$$\begin{aligned} a_{k+1} &= a_{k-1} + a_{k-2} \leq \left(\sqrt{2}\right)^{k-1} + \left(\sqrt{2}\right)^{k-2} \\ &= \left(\sqrt{2}\right)^{k-2}\left(\sqrt{2}+1\right) \leq 2\sqrt{2}\left(\sqrt{2}\right)^{k-2} = \left(\sqrt{2}\right)^{k+1}, \end{aligned}$$

**[1 pt]**

where the first inequality follows from the induction hypothesis applied to $p = k-1$ and $p = k-2$ respectively. **[1 pt]**

(From the proof it must be crystal clear whàt is supposed **[2 pt]** and whàt must be proved **[1 pt]**. In case the induction hypothesis is not correctly formulated or the proof is not clearly written down: at most **1 pt** for the entire exercise)

5. (a) (i) $f$ is not one-to-one. For example: $f(\{1\},\{1\}) = \varnothing = f(\{2\},\{2\})$, but $(\{1\},\{1\}) \neq (\{2\},\{2\})$. **[1 pt]**

(ii) $f$ is onto. For any $C \in \mathcal{P}(\{1,2,3\})$, we can take $A = C$ and $B = \varnothing$ then $f(A,B) = C - \varnothing = C$. **[2 pt]**

(iii) $f$ is not invertible, since $f$ is not one-to-one. **[1 pt]**

(b) $f^{-1}(\{1,3\}) = \{\, (\{1,3\},\varnothing),\ (\{1,2,3\},\{2\}),\ (\{1,3\},\{2\})\, \}$. **[2 pt]**

6. (a) (i) $R$ is reflexive since $ARA$ for all $A \in \mathcal{P}(U)$, because $A - D = A - D$. **[1 pt]**

(ii) $R$ is symmetric since for all $A, B \in \mathcal{P}(U)$, $ARB$ implies $BRA$, because if $A - D = B - D$, then $B - D = A - D$. **[1 pt]**

(iii) $R$ is transitive since for all $A, B, C \in \mathcal{P}(U)$, $ARB$ and $BRC$ imply $ARC$, because if both $A - D = B - D$ and $B - D = C - D$, then $A - D = C - D$. **[1 pt]**

(b) The partion of $A$ induced by $R$ consists of the equivalence classes of $R$. **[0.5 pt]**

So the partition is: $\{[\varnothing], [\{3\}], [\{4\}], [\{3,4\}]\}$, where

$[\varnothing] = \{\varnothing, \{1\}, \{2\}, \{1,2\}\}$;

$[\{3\}] = \{\{3\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$;

$[\{4\}] = \{\{4\}, \{1,4\}, \{2,4\}, \{1,2,4\}\}$;

$[\{3,4\}] = \{\{3,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$. **[2.5 pt]**