Discrete Mathematics for Computer Science, October 26, 2018; Part 1
Solution/Correction standard

1. (a) $$\forall x \forall y \left[x \leq y \rightarrow f(x) \geq f(y) \right].$$ **[2 pt]**

   (b) $$\forall x \left[0 \leq f(x) \leq 5 \right] \wedge \exists x \left[f(x) = 0 \right] \wedge \exists x \left[f(x) = 5 \right].$$ **[4 pt]**

   For each expression that is not logically equivalent to the ones above: 0 pt.

2.

(1)	$(p \rightarrow r) \rightarrow p$	Premise
(2)	$\neg(\neg p \vee r) \vee p$	(1), L12 (2$\times$)
(3)	$(p \wedge \neg r) \vee p$	(2), L2, L1
(4)	p	(3), L3, L10
(5)	$(p \rightarrow q) \wedge r$	Premise
(6)	$\neg p \vee q$	(5), R7, L12
(7)	$\neg\neg p$	(4), L1
(8)	q	(6),(7), R5
(9)	$\neg q \vee s$	Premise
(10)	$\neg\neg q$	(8), L1
(11)	s	(9),(10), R5
(12)	$s \wedge p$	(11),(4), R4
(13)	$\neg\neg s \wedge \neg\neg p$	(12), L1 (2$\times$)
(14)	$\neg(\neg s \vee \neg p)$	(13), L2
(15)	$\neg(s \rightarrow \neg p)$	(14), L12

[6 pt]

For each forgotten Law or Rule: -0.5 pt.
If deduction contains a step that is not logically correct: at most 1 pt for the entire exercise.

3. (a) Counterexample: $A = C = \{1\}$; $B = \varnothing$. **[1 pt]**
   Then $A - C = \varnothing = B - C$ and $A \cup C = B \cup C = \{1\}$. But $A \neq B$. **[1 pt]**

   (b) Suppose $A - C = B - C$ and $A \cap C = B \cap C$. We must show that $A = B$.
   We show that $A \subseteq B$ and $B \subseteq A$. **[0.5 pt]**

   (i) Proof of $A \subseteq B$.
   Let $x \in A$. We distinguish the cases $x \in C$ and $x \notin C$. **[0.5 pt]**
   Case 1: Suppose $x \in C$. Then $x \in A \cap C$. So $x \in B \cap C$, and hence $x \in B$. **[1 pt]**
   Case 2: Suppose $x \notin C$. Then $x \in A - C$. So $x \in B - C$. So again $x \in B$. **[1 pt]**
   From Case 1 and Case 2 we conclude $A \subseteq B$.

   (ii) Proof of $B \subseteq A$.
   This proof is analogous to part (i), by interhanging the roles of A and B. **[1 pt]**