

Reference : AM2017/DMMP/010/ha

Course : **Discrete Mathematics for Computer Science**

Date : October 27, 2017

Time : 08.45–09.45 hrs

**Motivate all your answers. The use of electronic devices is not allowed.**

In this exam:  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

1. [6 pt]

Let the sequence of integers  $a_0, a_1, a_2, \dots$  be given by:

$$a_0 = 1, a_1 = 2, a_2 = 7, \text{ and for } n \geq 3: a_n = 2a_{n-1} + a_{n-2} + 2a_{n-3}.$$

Prove with mathematical induction that for all  $n \in \mathbb{N}$ ,  $a_n \leq \left[\frac{8}{3}\right]^n$ .

2. [6 pt]

Let  $A, B$  and  $C$  be sets and let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be functions such that  $f$  is onto and  $g \circ f$  is one-to-one. Prove that  $g$  is one-to-one.

3. [6 pt]

Let  $A = \{(x, y) \mid x, y \in \{0, 1, 2\}\}$  and let  $R$  be the relation on  $A$  given by:

$$(a, b)R(c, d) \quad \text{if and only if} \quad a + d = b + c.$$

Show that  $R$  is an equivalence relation on  $A$  and determine the partition of  $A$  induced by  $R$ .

**Total:** 18 points