

**Discrete Mathematics for Computer Science  
Part 2, Sample Test 2**

**Duration: 60 min.**

**Motivate all your answers.**

**The use of electronic devices is not allowed.**

In this exam:  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

1. [6 pt]

Prove with mathematical induction that for all  $n \in \mathbb{N}$ ;  $n \geq 60$ ,  
 $n$  can be written as  $n = 6s + 13t$ , for some  $s, t \in \mathbb{N}$ .

2. [6 pt]

Let  $A$  and  $B$  be sets,  $f : A \rightarrow B$  a function and let  $B_1, B_2 \subseteq B$ .

Prove that:  $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$ .

3. [6 pt]

Let  $A = \mathbb{R}^+ \times \mathbb{R}^+$  and let  $R$  be the relation on  $A$  given by:

$(x_1, y_1)R(x_2, y_2)$  if and only if  $x_1y_2 = x_2y_1$ .

Show that  $R$  is an equivalence relation on  $A$  and describe the partition of  $A$  induced by  $R$ .

**Total: 18 points**