

Discrete Mathematics for Computer Science

Part 2, Sample Test 1

Duration: 60 min.

Motivate all your answers.

The use of electronic devices is not allowed.

In this exam: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$.

1. [6 pt]

Let the sequence of numbers $a_1, a_2, a_3, a_4, \dots$ be given by:

$$a_1 = 2, a_2 = 6, a_3 = 15, \text{ and for } n \geq 4: \quad a_n = a_{n-1} + 2a_{n-2} + 4a_{n-3}.$$

Prove with mathematical induction that for all $n \in \mathbb{Z}^+$, $a_n \leq \left[\frac{5}{2}\right]^n$.

2. [6 pt]

Let \mathcal{U} be a nonempty universe and let $f : \mathcal{P}(\mathcal{U}) \times \mathcal{P}(\mathcal{U}) \rightarrow \mathcal{P}(\mathcal{U})$ be the operation on $\mathcal{P}(\mathcal{U})$ given by

$$f(A, B) = \overline{A \cup B}.$$

Examine if f is commutative, associative and if f has an identity.

3. [6 pt]

Let A be a finite set and let R be a relation on A . Let M be the relation matrix for R .

Prove that: R is transitive if and only if $M^2 \leq M$.

Total: 18 points