

**Discrete Mathematics for Computer Science**  
**Part 1; Sample Test 2**

**Duration: 60 min.**

**Motivate all your answers. The use of electronic devices is not allowed.**  
**A formula sheet is included.**

In this exam:  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

1. Consider a sequence of  $n$  integers  $X(n) = (x_1, x_2, \dots, x_n)$  ( $x_i \in \mathbb{Z}$ ).  
Give quantified expressions for the following statements.
  - (a) [2 pt] The sequence  $X(n)$  is decreasing.  
(an example of a decreasing sequence of 5 integers is  $(8, 7, 7, 2, -4)$ )
  - (b) [4 pt] The greatest integer in  $X(n)$  is 10.  
(e.g. the greatest integer in the 4-integer sequence  $(3, 5, -1, 5)$  is 5)
  
2. [6 pt]  
Prove the validity of the following argument using the "Laws of Logic", the "Rules of Inference" and the supplement w.r.t. quantifiers (the N-Laws and U-Rules).

$$\frac{\forall x [p(x) \rightarrow q(x)] \quad \exists x [p(x) \vee q(x)]}{\therefore \exists x q(x)}$$

3. [6 pt]  
Let  $A$  and  $B$  be sets in a universe  $\mathcal{U}$ .  
Prove that:  $\overline{A \Delta B} = A \Delta B$ .

**Total: 18 points**