

Discrete Mathematics for Computer Science,
Part 1; Sample Test 2. Solutions/Correction standard

1. (a) $\forall_{k \in \{1, \dots, n-1\}} [x_k \geq x_{k+1}]$. [2 pt]

(b) $\forall_{j \in \{1, \dots, n\}} [x_j \leq 10] \wedge \exists_{k \in \{1, \dots, n\}} [x_k = 10]$. [4 pt]

For each expression that is not logically equivalent to the ones above: 0 pt.

2.

(1)	$\exists x [p(x) \vee q(x)]$	Premise
(2)	$p(c) \vee q(c)$ for some $c \in \mathcal{U}$	(1), U2
(3)	$\forall x [p(x) \rightarrow q(x)]$	Premise
(4)	$p(c) \rightarrow q(c)$	for the c in (2) (3), U1
(5)	$\neg p(c) \vee q(c)$	for the c in (2) (4), L11
(6)	$[p(c) \vee q(c)] \wedge [\neg p(c) \vee q(c)]$	for the c in (2) (2),(5), R4
(7)	$[q(c) \vee p(c)] \wedge [q(c) \vee \neg p(c)]$	for the c in (2) (6), L3 (2×)
(8)	$q(c) \vee [p(c) \wedge \neg p(c)]$	for the c in (2) (7), L5
(9)	$q(c) \vee F_0$	for the c in (2) (8), L8
(10)	$q(c)$	for the c in (2) (9), L7
(11)	$\exists x q(x)$	(10), U4

[6 pt]

For each forgotten Law or Rule: -0.5 pt.

If deduction contains a step that is not logically correct: at most 1 pt for the entire exercise.

Remark: Also R10 can be used, yielding $(p(c) \vee \neg p(c)) \rightarrow q(c)$, and then applying L11, L2, L1, L3, L8, L3, L7 and U4 respectively yields $\exists x q(x)$.

3. (i) Proof of $\overline{A \Delta B} \subseteq A \Delta B$.

Let $x \in \overline{A \Delta B}$. Then $x \in \overline{A \cup B}$ and $x \notin \overline{A \cap B}$.

Case 1. If $x \in A$, then $x \in A \cup B$. Also $x \notin \overline{A}$, so $x \in \overline{B}$ (because $x \in \overline{A \cup B}$). So $x \notin B$, and therefore $x \notin A \cap B$. Now $x \in A \cup B$ and $x \notin A \cap B$ imply $x \in A \Delta B$.

Case 2. If $x \notin A$, then $x \in \overline{A}$. So $x \notin \overline{B}$ (because $x \notin \overline{A \cap B}$). So $x \in B$ and hence $x \in A \cup B$. Also $x \notin \overline{B}$ implies $x \notin \overline{A \cap B}$. Hence $x \in A \Delta B$. [3 pt]

(ii) Proof of $A \Delta B \subseteq \overline{A \Delta B}$.

Let $x \in A \Delta B$. Then $x \in A \cup B$ and $x \notin A \cap B$.

Case 1. If $x \in A$, then $x \notin B$ (because $x \notin A \cap B$). So $x \in \overline{B}$ and therefore $x \in \overline{A \cup B}$. Also $x \in A$ implies $x \notin \overline{A}$ and so $x \notin \overline{A \cap B}$. Hence $x \in \overline{A \Delta B}$.

Case 2. If $x \notin A$, then $x \in \overline{A}$. So $x \in \overline{A \cup B}$. Also $x \in B$ (because $x \in A \cup B$ and $x \notin A$). So $x \notin \overline{B}$ and so $x \notin \overline{A \cap B}$. So $x \in \overline{A \Delta B}$. [3 pt]