Discrete Mathematics for Computer Science, Part 1; Sample Test 1. Solution/Correction standard

1. (a) $\exists_{a\in A}\exists_{k\in\mathbb{Z}^+}\left[a=k^2\right]$. **[3 pt]**

   (b) $\neg\exists_{a\in A}\left[a\neq 1\wedge\forall_{d\in\mathbb{Z}^+}\left[d\,|\,a\to(d=1\vee d=a)\right]\right]$. **[3 pt]**

   For each expression that is not logically equivalent to the ones above: **[0 pt]**

2. We take $\neg t$ as extra premise and prove: $\neg q$.

(1)	$\neg t$	Extra Premise
(2)	$p\to t$	Premise
(3)	$\neg t\to\neg p$	(2), L13
(4)	$\neg p$	(1),(3), R1
(5)	$\neg p\vee q$	(4), R8
(6)	$(\neg p\vee q)\to r$	Premise
(7)	r	(5),(6),R1
(8)	$\neg p\wedge r$	(4),(7),R4
(9)	$(\neg p\wedge r)\to\neg s$	Premise
(10)	$\neg s$	(8),(9),R1
(11)	$s\vee\neg q$	Premise
(12)	$\neg q$	(11),(10), R5

[6 pt]

For each forgotten Law or Rule: -1 **pt**.
If deduction contains a step that is not logically correct: at most **1 pt** for the entire exercise.

3. (a) Let $C\in\mathcal{P}(A)\cup\mathcal{P}(B)$. Then $C\in\mathcal{P}(A)$ or $C\in\mathcal{P}(B)$, so $C\subseteq A$ or $C\subseteq B$. Hence $C\subseteq A\cup B$, and so $C\in\mathcal{P}(A\cup B)$. **[3 pt]**

   (b) The statement is false.
   Counterexample: $\mathcal{U}=\{1,2\}$, $A=\{1\}$ and $B=\{2\}$. **[1 pt]**
   Then $A\cup B=\{1,2\}$, $\mathcal{P}(A)=\{\varnothing,\{1\}\}$ and $\mathcal{P}(B)=\{\varnothing,\{2\}\}$.
   So $\mathcal{P}(A\cup B)=\{\varnothing,\{1\},\{2\},\{1,2\}\}$ and $\mathcal{P}(A)\cup\mathcal{P}(B)=\{\varnothing,\{1\},\{2\}\}$. **[2 pt]**