

Discrete Mathematics: Additional Exercises to Lecture 4

1. Prove the validity of the following argument using the "Laws of Logic", the "Rules of Inference" and the supplement w.r.t. quantifiers (the N-Laws and U-Rules).

$$\begin{array}{l} \forall x [p(x) \rightarrow (q(x) \vee r(x))] \\ \exists x p(x) \\ \forall x [q(x) \rightarrow r(x)] \\ \hline \therefore \exists x r(x) \end{array}$$

2. Show, by giving a counterexample, that the following argument is invalid.

$$\begin{array}{l} \forall x [p(x) \vee q(x)] \\ \forall x [p(x) \vee r(x)] \\ \forall x [q(x) \vee r(x)] \\ \hline \therefore (\forall x p(x) \vee \forall x q(x) \vee \forall x r(x)) \end{array}$$