

Discrete Mathematics: Solutions Additional Exercises to Lecture 4

1.

(1)	$\exists x p(x)$		Premise
(2)	$p(c)$	for some $c \in \mathcal{U}$	(1), U2
(3)	$\forall x [p(x) \rightarrow (q(x) \vee r(x))]$		Premise
(4)	$p(c) \rightarrow (q(c) \vee r(c))$	for the c in (2)	(3), U1
(5)	$q(c) \vee r(c)$		(2),(4), R1
(6)	$\forall x [q(x) \rightarrow r(x)]$		Premise
(7)	$q(c) \rightarrow r(c)$	for the c in (2)	(1), U2
(8)	$\neg q(c) \vee r(c)$		(7), L12
(9)	$r(c) \vee \neg q(c)$		(8), L3
(10)	$r(c) \vee q(c)$		(5), L3
(11)	$(r(c) \vee q(c)) \wedge (r(c) \vee \neg q(c))$		(10),(9), R4
(12)	$r(c) \vee (q(c) \wedge \neg q(c))$		(11), L5
(13)	$r(c) \vee F_0$		(12), L8
(14)	$r(c)$		(13), L7
(15)	$\exists x r(x)$		(14), U4

2. E.g. take $\mathcal{U} = \mathbb{N}$ and

$$p(x) : x \neq 1,$$

$$q(x) : x \neq 2,$$

$$r(x) : x \neq 3.$$

Then all premises are true, but the conclusion is false.