Homework Lecture 4 (solutions)

3.1

```
  1 0 1 1 0        1 1 1 1 0        1 1 1 1 1
+ 1 0 1 1 1      + 1 1 1 0 1      + 0 1 1 1 1
-----------      -----------      -----------
  0 1 1 0 1        1 1 0 1 1        0 1 1 1 0
```

Overflow | No overflow | No overflow

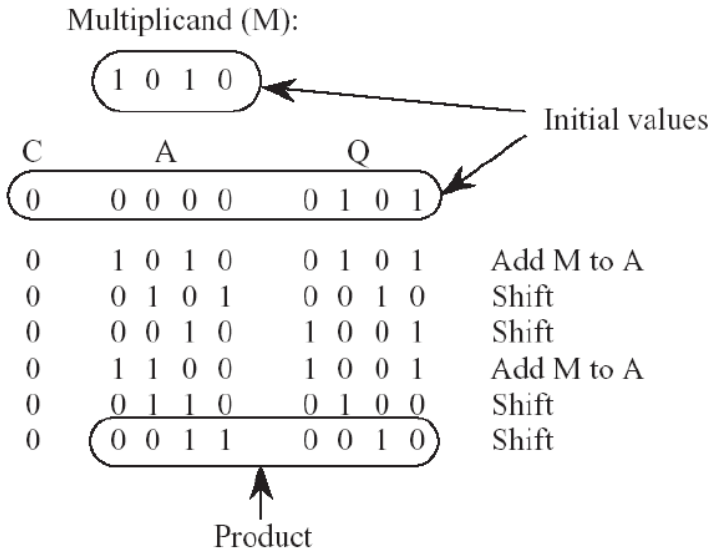
3.3

```
Two's complement
  1 0 1 1.1 0 1
+ 0 1 1 1.0 1 1
---------------
  0 0 1 1.0 0 0 (no overflow)

One's complement
  1 0 1 1.1 0 1
+ 0 1 1 1.0 1 1
---------------
  0 0 1 1.0 0 1 (no overflow)
```

Note that for the one's complement solution, that the end-around carry is added into the 1's position.

3.4



Exercise 1

Solution 1:
Operand 1: $1011_2 = -5 \rightarrow 5 = 0101$
Operand 2: $0110_2 = 6$
Sign are different; perform unsigned multiplication 0101 × 0110

```
   0101
   0110 ×
   0000
  0101
 0101
0000      +
00011110
```

Signs of operand different→twos complement of result → 11100010

Solution 2: (sign extension)

```
     11111011
     00000110 ×
    111110110
   111110110  +
  101111100010
```

Exercise 2

a) M_{max} 0.1 followed by 23 ones = $1-2^{-24} \sim 0.99999$

b) M_{min} 0.1 followed by 23 zeros = 0.50000

c) E_{max} $2^8-1-128 = 127$

d) E_{min} $1-128 = -127$

e) V_{max} $(1-2^{-24}) \times 2^{127} \sim 0.170141 \times 10^{39}$

f) V_{min} $2^{-1} \times 2^{-127} = 2^{-128} \sim 0.293874 \times 10^{-39}$

g) smallest positive number: $2^{-1} \times 2^{-127}$
next positive number: $(2^{-1} + 2^{-24}) \times 2^{-127}$
The Δr between zero en smallest positive number is $2^{-1} \times 2^{-127} = 2^{-128}$
The Δr smallest positive number and next positive number $2^{-24} \times 2^{-127} = 2^{-151}$
This is not a nice property of the DEC format. In the IEEE Floating Point due the denormalized numbers this effect does not occur.

h) Representation of $2\ ^9/_{16}$?
$2\ ^9/_{16} = {}^{41}/_{16}$
$0.5 <= \text{mantissa} < 1$
${}^{41}/_{16} = {}^{41}/_{16} \times {}^4/_4 = {}^{41}/_{64} \times 2^2$
Mantissa is 0.1010010000000 => bit pattern: 01001000....0
Exponent is 2 => in excess 128 is 130 => bit pattern: 10000010
Sign positive => bit: 0

i) Representation of the decimal value 0.2 ?
0.2
$0.5 <= \text{mantissa} < 1$
${}^2/_{10} = {}^2/_{10} \times {}^4/_4 = {}^8/_{10} \times 2^{-2}$
Mantissa is $({}^8/_{10} \times 2^{24)} \times 2^{-24} = 13421772.8 \times 2^{-24}$
Value can not be exactly represented! Round to 13421772×2^{-24}
0.11001100110011001100, the first 0.1 is hidden so the bit pattern for the mantissa is: => 10011001100110011001100

Exponent is -2 => in excess 128 is 126 => bit pattern: 11111110
Sign positive => bit: 0

j) Pattern:
1 00000111 11010000000000000000001

sign: negative
Exponent: 00000111_2 = unsigned representation 7_{10} =
excess 128: 7-128= -121_{10}
Mantissa: **0.1**11010000000000000000001
$2^{-1} + 2^{-2} + 2^{-3} + 2^{-5} + 2^{-24} \sim 0.90625006$

Decimal value represented is $-0.90625006 \times 2^{-121} \sim 3.4089338 \times 10^{-37}$

Exercise 3

Double precision IEEE FP standard

a) M_{max} 1. followed by 52 ones = $2-2^{-52} \sim 1.99999$

b) M_{min} 1. followed by 52 zeros = 1

c) E_{max} $2^{11}-1-1-1023 = 1023$

d) E_{min} $1- 1023 = -1022$

e) V_{max} $(2-2^{-52}) \times 2^{1023}$

f) V_{min} 1×2^{-1022}