

Homework Lecture 3 (solutions)

2.2 (a) 101111

(b) 111011

(c) 531

(d) 22.625

(e) 202.22

2.3 (a) 27

(b) 000101

(c) 1B

(d) 110111.111

(e) 1E.8

2.4 $2 \times 3^{-1} + 0 \times 3^{-2} + 1 \times 3^{-3} = 2/3 + 0 + 1/27 = 19/27$

2.5 37.3

$$(43.3)_7 = (4 \times 7^1 + 3 \times 7^0 + 3 \times 7^{-1})_{10} = 31.4287 \dots_{10}$$

$$31_{10} = \mathbf{37}_8$$

$$.4287 \dots_{10} = (\mathbf{3.4296 \dots}) \times 8^{-1}$$

$$\rightarrow (37.3)_8$$

2.6

17.5

Base 3 \rightarrow 122.1111111

Two digits precision

$122.11_3 \rightarrow 17.44_{10}$

2.7 -8

2.8 0

2.9

One's complement has two representations for zero, whereas two's complement has one representation for zero, thus two's complement can represent one more integer.

2.15

Sign bit: 0 (positive)

exp.bits: 7 bits in excess 64

Fraction: 24 bits (no hidden bit)

Base 2

Truncation is used

107.15_{10}

$107_{10} = 1101011_2$

$0.15 \times 2 \Rightarrow 0.3 \quad 0$

$0.3 \times 2 \Rightarrow 0.6 \quad 0$

$0.6 \times 2 \Rightarrow 1.2 \quad 1$

$0.2 \times 2 \Rightarrow 0.4 \quad 0$

$0.4 \times 2 \Rightarrow 0.8 \quad 0$

$0.8 \times 2 \Rightarrow 1.6 \quad 1$

$0.6 \times 2 \Rightarrow 1.2 \quad 1$

Repeating pattern

Pattern is: 0010011001100110011...

$107_{10} = 1101011.0010011001100110011...._2$

Normalization: point 7 positions to the left \rightarrow increase exponent with 7

$7 + 64 = 71 \Rightarrow 1000111_2$

Floating point format:

0 1000111 110101100100110011001100

- 2.16** (a) $+1.011 \times 2^4$
(b) -1.0×2^1
(c) -0
(d) $-\infty$
(e) $+\text{NaN}$
(f) $+1.1001 \times 2^{-126}$
(g) $+1.01101 \times 2^{-124}$

Note: the author has chosen implicitly base 2 for the mantissa 1.011; therefore answer a) is $+(1.011)_2 \times 2^4$

In decimal the answer would be:

$$(11/8) \times 2^4 =$$
$$2.2 \times 2^4$$

- 2.17** (a) 0 10000100 1011 0000 0000 0000 0000 000
 (b) 0 00000000 0000 0000 0000 0000 0000 000
 (c) 1 0111111110 0011 1000 0000 0000 0000
 0000 0000 0000 0000 0000 0000 0000 0000
 (d) 1 11111111 1011 1000 0000 0000 0000 000

Remark 2.17d)

For NaN the exponent field is 11111111 and the fraction is not equal to zeros. The author has chosen the pattern 1011 1000..000 (without explanation). In the floating point standard (std 754) it is mentioned that NaN may carry a payload (fraction field) that is intended for diagnostic information indicating the source of the NaN.

This is not part of this course.

- 2.18** (a) $(2 - 2^{-23}) \times 2^{127}$
 (b) 1.0×2^{-126}
 (c) $2^{-23} \times 2^{-126} = 2^{-149}$
 (d) $2^{-23} \times 2^{-126} = 2^{-149}$
 (e) $2^{-23} \times 2^{127} = 2^{104}$
 (f) $2 \times (127 - -126 + 1) \times 1 \times 2^{23} + 1 = 254 \times 2^{24} + 1$