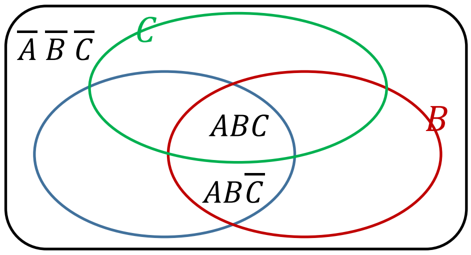
Solutions Exercises Chapter 1

1. **a.** $AB\overline{C}$ **b.** ABC **c.** $A \cup B \cup C$ **d.** $AB \cup AC \cup BC$
e. $\overline{A}\,\overline{B}\,\overline{C} = \overline{A \cup B \cup C}$ **f.** $A\overline{B}\,\overline{C} \cup \overline{A}B\overline{C} \cup \overline{A}\,\overline{B}C$ **g.** $\overline{ABC} = \overline{A} \cup \overline{B} \cup \overline{C}$ (De Morgan)

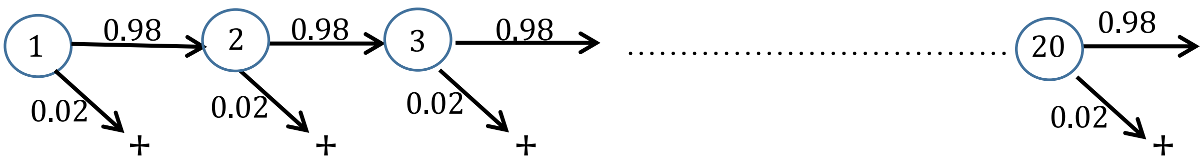
Exercise 1 illustrated with a Venn-diagram:
- the area`of parts a., b. and e. are indicated
- In part e. one can reason: if an outcome in in none of the three events, it is outside union of the three events, so part of the complement of $A \cup B \cup C$.



2. At least 2 out of three means $AB\overline{C}$ or $A\overline{B}C$ or $\overline{A}BC$ or ABC.
Use the Venn-diagram of three events above to see that the event, that A ànd B ànd C occur (so ABC), 300 bolts. Furthermore 400 bolts are in A and B. Since AB can be slit up in ABC and $AB\overline{C}$ (a partition of AB !), $AB\overline{C}$ has to consist of 100 bolts. Likewise we find that $A\overline{B}C$ or $\overline{A}BC$ consists of 100 bolts as well: in total there are $100 + 100 + 100 + 300$ bolts in at least 2 out of 3 events: the probability is $\frac{600}{1200}$

3. By distinguishing the coins (choose a 1€ and a 2€ coin), it is clear that we have 4 equally likely outcomes: $(K, K), (K, M), (M, K)$ en (M, M).
2 of these 4 outcomes occur as the event "one Tails and one Heads".

4. The statement is **not correct**. According to the same reasoning the probability of being shot down in 100 flights would be 200%, which is evidently incorrect. The reasoning causes "double counting". The probability might be interpreted as the probability to be shot down at least one time. The probability of the complement "not to be shot down in 20 flights" is 0.98^{20}. So the probability to be shot down is not 40% but $1 - 0.98^{20} \approx 33.2\%$. The correct reasoning can be illustrated by the following "probability tree" (a circle is a flight and a "+" means "shot down"):



The requested probability can also be computed from this diagram, as a sum of probabilities: $0.02 + 0.98 \cdot 0.02 + 0.98^2 \cdot 0.02 + \cdots + 0.98^{19} \cdot 0.02$,
but our first solution with the complement rule is easier
P("shot down in 20 flights") = 1 – P("not shot down in 20 flights")

5. Use the Venn-diagram of exercise again to see that, if you add the probabilities $P(A), P(B)$ and $P(C)$ you will be double counting: if we would subtract ($P(AB), P(BC)$ and $P(AC)$), we are excluding the intersection of the three events: to compensate we add $P(ABC)$ once. So:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC)$$

Note: Just stating this property using the Venn-diagram is sufficient for this applied course. Students in mathematics should give a more formal prove of the statement, such as:

$$P(A \cup B) = P(A) + P(B) - P(AB).$$ Replace B by $B \cup C$, then:

$$P\big(A \cup (B \cup C)\big) = P(A) + P(B \cup C) - P\big(A \cap (B \cup C)\big)$$

Since $P(B \cup C) = P(B) + P(C) - P(BC)$

and $P\big(A \cap (B \cup C)\big) = P(AB \cup AC) = P(AB) + P(AC) - P(ABC)$, we have:

$$P(A \cup B \cup C) = P(A) + [P(B) + P(C) - P(BC)] - [P(AB) + P(AC) - P(ABC)]$$
$$= P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC).$$ **QED**

6. **a.** 1 dice is enough, e.g.: "even" = a and "odd" = b.

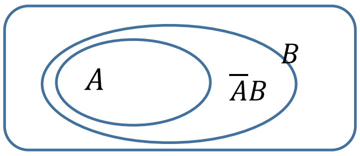
b. 2 dice: $6 \cdot 6 = 36$ pairs of outcomes, which ae equally likely. Since 36 can be divided by 4, it works. One could choose a. if both dice are at most 3: probability $\frac{3 \cdot 3}{36}$.

c. Not possible, since rolling n dice will give 6^n equally likely outcomes, but 6^n cannot be divided by 5 to create 5 equally likely events.

(*One could roll one dice and award the outcomes 1 to 5 to the answers a. to e. And if you roll 6 we will repeat the procedure. But in this way we do not have a fixed number of rolls).*

7. $A \subset B$ means: $B = A \cup (\bar{A}B)$, where A and $\bar{A}B$ are mutually exclusive (axiom 3): $P(B) = P\big(A \cup \bar{A}B\big) = P(A) + P\big(\bar{A}B\big)$.

Since $P\big(\bar{A}B\big) \geq 0$ (axiom 1), we find: $P(B) \geq P(A)$

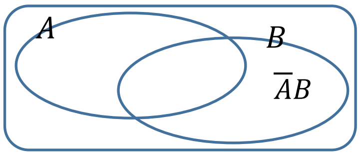


8. Use the Venn-diagram to see that $P(A \cup B) = P(A) + P\big(\bar{A}B\big)$

So $P\big(\bar{A}B\big) = P(A \cup B) - P(A) = \frac{8}{9} - \frac{1}{2} = \frac{7}{18}$

But then $P(B) = P(AB) + P\big(\bar{A}B\big) = \frac{1}{3} + \frac{7}{18} = \frac{13}{18}$ and

$$P\big(\bar{A} \cap \bar{B}\big) = 1 - P(A \cup B) = 1 - \frac{8}{9} = \frac{1}{9}$$



(Follows from the Venn-diagram or De Morgan`s rule: $P\big(\bar{A} \cap \bar{B}\big) = P\big(\overline{A \cup B}\big)$)