# Self-study, week 6

1. For the second order differential equation

$$y'' + 5y' + 4y = 0$$

show that:

1.1 For all $c_1, c_2 \in \mathbb{R}$

$$c_1 e^{-x} \text{ and } c_2 e^{-4x}$$

are solutions as well.

1.2 For all $c_1, c_2 \in \mathbb{R}$

$$c_1 e^{-x} + c_2 e^{-4x}$$

is a solution.

2. Consider the second order differential equation

$$y'' + 3y' + 2y = 0.$$

2.1 Solve this equation when $y(0) = 0$ and $y'(0) = -1$

2.2 Sketch the above solution.

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3. In this exercise we take a closer look at the case when the two zeros of the characteristic equation are the same.
   - 3.1 Find all functions $f$ which satisfy $f'' = 0$.
   - 3.2 Consider the second order differential equation

$$y'' + 2y' + y = 0. \tag{1}$$

     Determine the zeros of the characteristic equation. Next define $z(x) = y(x)e^x$ and show that $z'' = 0$.
   - 3.3 Find all solutions of (1).
4. Solve the differential equations
   - 4.1 $y'' + 4y = \sin(x)$
   - 4.2 $y'' - y = x$