

## Self-study, week 5

1. Calculate for  $z_1 = 3 + 4i$ ,  $z_2 = -7 + 11i$  the following expressions, and write them in the form  $a + ib$ , with  $a, b$  real

1.1  $\overline{z_1}$

1.2  $|z_1|$

1.3  $z_1 + 3z_2$

1.4  $z_1 - i \cdot z_2$

1.5  $z_1 z_2$

1.6  $\frac{z_2}{z_1}$

2. Use Euler's formula to prove:

2.1  $\overline{e^{i\theta}} = e^{-i\theta}$

2.2  $|e^{i\theta}| = 1$

2.3  $\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$

2.4  $\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

3. Use De Moivre's Theorem to prove that

▶  $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$

▶  $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

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4. Determine all  $\theta \in \mathbb{R}$  for which the following equality holds:

$$e^{i\theta} = 1.$$

5. Prove that for  $z = a + bi \in \mathbb{C}$ , the following relation holds

$$(e^{zt})' = ze^{zt}.$$

Here the prime denotes the derivative with respect to the real variable  $t$ .

**Remark:** For the function  $f = f_{Re} + if_{Im}$  with  $f_{Re}, f_{Im}$  real and imaginary part of the function  $f(t)$  respectively, the derivative equals

$$f' = f'_{Re} + if'_{Im}.$$