Self Study exercises Math B2, week 3

1. Consider the basic power series $\sum_{n=0}^{\infty} x^n$ which is divergent for $|x| \geq 1$ and convergent for $-1 < x < 1$ (with sum $\frac{1}{1-x}$).

   Find the series' radius and interval of convergence; in case of convergency, find the sum.

   (a) $\sum_{n=0}^{\infty}(x+5)^n$

   (b) $\sum_{n=0}^{\infty}(2x)^n$

2. The series $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \ldots$ converges to $\sin(x)$ for all x.

   Find a series that converges to $\cos(2x)$ for all x. (use the term-by-term differentiation, see Thomas page 598.)

3. Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by $f(x) = \frac{1}{x+2}$ at $a = 0$.

   (for the definition, see Thomas, page 604.)

4. Find the Maclaurin series for the function $\sinh(x) = \frac{e^x - e^{-x}}{2}$. (instead of using the definition, the series for e^x (see page 604) can help you to explore the sine-hyperbolic function.)

5. Find the sum of the following series:

   (a) $\sum_{n=0}^{\infty} \frac{x^{4n}}{n!}$

   (b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^{2n+1}(2n+1)!}$

   (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n \cdot n!}$