Self Study exercises Math B2, week 2

1. Evaluate the following indefinite integrals

   (a) $\int \frac{4x^3}{(x^4+1)^2}\ \mathrm{d}x$ (substitute $u = x^4 + 1$)

   (b) $\int \frac{1}{5s+4}\ \mathrm{d}s$ (substitute $u = ...$)

2. See Thomas, page 344, for the substitution formula in definite integrals (the limits of integration change by substitution!). Use this formula in evaluating the following definite integrals:

   (a) $\int_0^1 r\sqrt{1-r^2}\ \mathrm{d}r$ (substitute $u = 1 - r^2$)

   (b) $\int_0^{\frac{\pi}{3}} \frac{4\sin\theta}{1-4\cos\theta}\ \mathrm{d}\theta$ (substitute $u = ...$)

3. See Thomas, page 346. Theorem 8 deals with the definite integral of symmetric functions.

   (a) Show that if f is odd on $[-\pi, \pi]$, then $\int_{-\pi}^{\pi} f(x)\ \mathrm{d}x = 0$.

   (b) Find $\int_{-\pi}^{\pi} \sin x \cos x\ \mathrm{d}x$ and $\int_{-\pi}^{\pi} \sin^2 x\ \mathrm{d}x$.

4. Use the formula $\int u\ \mathrm{d}v = uv - \int v\ \mathrm{d}u$ (integration by parts) in evaluating the following integrals:

   (a) $\int x\ln(x)\ \mathrm{d}x$ (take $u = \ln(x)$ and $v = \frac{1}{2}x^2$.)

   (b) $\int xe^{4x}\ \mathrm{d}x$

   (c) $\int x^2e^{4x}\ \mathrm{d}x$

5. See Thomas, page 458, integration by parts for definite integrals.

   Find $\int_0^{\pi} t\sin(t)\ \mathrm{d}t$.

6. Evaluate $\int_0^4 \frac{\mathrm{d}x}{\sqrt{4-x}}$

   (improper integral, use limit for evaluation, see Thomas page 504.)