Exercise Lecture 6 slide 87

$$y'' - y' - 2y = -\cos(x) - 3\sin(x)$$

- First solve general homogeneous solution y_H

$y'' - y' - 2y = 0$

$r^2 - r - 2 = 0 \iff (r-2)(r+1) = 0 \iff$

$r = 2$ or $r = -1$

$[\, y_H = c_1 e^{2x} + c_2 e^{-x}$ with $c_1, c_2 \in \mathbb{R}$

- Next, try to find a particular solution

$$\left.\begin{aligned} y_p &= (\text{try}) \quad \alpha\cos(x) + \beta\sin(x) \\ y_p' &= -\alpha\sin(x) + \beta\cos(x) \\ y_p'' &= -\alpha\cos(x) - \beta\sin(x) \end{aligned}\right\} \text{substitute in ODE}$$

r.h.s. $-\alpha\cos(x) - \beta\sin(x) + \alpha\sin(x) - \beta\cos(x) - 2\alpha\cos(x) - 2\beta\sin(x) =$

$\cos(x)\cdot(-\alpha - \beta - 2\alpha) + \sin(x)\cdot(-\beta + \alpha - 2\beta)$

l.h.s. $-\cos(x) - 3\sin(x) \Rightarrow \begin{cases} -3\alpha - \beta = -1 \\ -3\beta + \alpha = -3 \end{cases}$

$-3(-3\alpha + 1) + \alpha = -3 \Rightarrow 10\alpha - 3 = -3 \Rightarrow \alpha = 0,\ \beta = 1$

$[\, y_p = \sin(x)$ $\quad y(0) = 1 \quad y'(0) = 3$

- Denote the general inhomogeneous solution, use conditions

$[\, y = c_1 e^{2x} + c_2 e^{-x} + \sin(x)$

$y' = 2c_1 e^{2x} - c_2 e^{-x} + \cos(x)$

$y(0) = c_1 + c_2 = 1$, $\quad c_1 + c_2 = 1$, $\quad 3c_1 = 3$

$y'(0) = 2c_1 - c_2 + 1 = 3$, $\quad 2c_1 - c_2 = 2$, $\quad c_1 = 1$

$c_2 = 0$

- Write down the answer

Solution: $[\, y = e^{2x} + \sin(x)$