Compute

$$\int_0^\infty \frac{x}{(1+x^2)^2}\,dx = \lim_{t\to\infty} \int_0^t \frac{x}{(1+x^2)^2}\,dx,$$ first

$$\int \frac{x}{(1+x^2)^2}\,dx = \int \frac{\frac{1}{2}\,dx^2}{(1+x^2)^2} \underset{\uparrow}{=} \int \frac{\frac{1}{2}\,du}{(1+u)^2} =$$

subst $u = x^2$

$$-\frac{1}{2}\,(1+u)^{-1} = -\frac{1}{2}(1+x^2)^{-1}$$

$$\int_0^t \frac{x}{(1+x^2)^2}\,dx = \left[-\frac{1}{2}(1+x^2)^{-1}\right]_0^t = -\frac{1/2}{1+t^2} + \frac{1/2}{1}$$

$$\lim_{t\to\infty}\left(-\frac{1/2}{1+t^2} + \frac{1/2}{1}\right) = +1/2,$$ so $\int_0^\infty \frac{x}{(1+x^2)^2}\,dx = \frac{1}{2}$, convergent

Partial integration: $\int u\,dv = uv - \int v\,du$

Find $\int x \ln^2(x)\,dx \underset{\uparrow \text{part. int.}}{=} \int \ln^2(x)\, d\left(\frac{1}{2}x^2\right) =$

$\frac{1}{2}x^2 \ln^2(x) - \int \frac{1}{2}x^2\, d\ln^2(x) =$

$\frac{1}{2}x^2 \ln^2(x) - \int \frac{1}{2}x^2 \cdot 2 \cdot \ln(x) \cdot \frac{1}{x}\,dx =$

$\boxed{\frac{1}{2}x^2 \ln^2(x)} - \boxed{\int x \ln(x)\,dx}$ remains to

compute $\int x \ln(x)\,dx \underset{\uparrow \text{part. int.}}{=} \int \ln(x)\, d\left(\frac{1}{2}x^2\right) =$

$\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2\, d\ln(x) =$

$\frac{1}{2}x^2 \ln(x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + C$

Answer $\frac{1}{2}x^2 \ln^2(x) - \frac{1}{2}x^2 \ln(x) + \frac{1}{4}x^2 + C$