Solutions Test Mathematics A; September 22, 2017.

1. (a) $A = [k, 50+k)$.

   (b) $$\bigcup_{k=1}^{50} A_k = [1,100), \qquad \bigcap_{k=1}^{50} A_k = [50,51).$$

2. $$\forall x[(x \in B \wedge x \in C) \to x \notin A]$$

3. (a) There exist $k, \ell \in \mathbb{Z}$ with $a = 2k$ and $b = 2\ell + 1$.
   Then
   $$a - b = 2k - (2\ell+1) = 2(k-\ell-1)+1.$$
   So $a - b$ is odd.

   (b) Basis step for $n = 1$:
   $$\sum_{i=1}^{1}(-1)^i \cdot i^2 = (-1)^1 \cdot 1^2 = -1 \quad \text{and also} \quad \frac{(-1)^1 \cdot 1 \cdot (1+1)}{2} = -1.$$
   So the statement is correct for $n = 1$.

   Induction step:

   Let $k \geq 1$ and suppose that:
   $$\sum_{i=1}^{k}(-1)^i \cdot i^2 = \frac{(-1)^k \cdot k(k+1)}{2}. \quad \text{(Induction hypothesis: IH)}$$
   We must show that IH implies: $\displaystyle\sum_{i=1}^{k+1}(-1)^i \cdot i^2 = \frac{(-1)^{k+1} \cdot (k+1)(k+2)}{2}$.

   Well: $\displaystyle\sum_{i=1}^{k+1}(-1)^i \cdot i^2 = \sum_{i=1}^{k}(-1)^i \cdot i^2 + (-1)^{k+1} \cdot (k+1)^2$.

   By IH this expression is equal to $\displaystyle\frac{(-1)^k \cdot k(k+1)}{2} + (-1)^{k+1} \cdot (k+1)^2$.
   Now it remains to show that
   $$\frac{(-1)^k \cdot k(k+1)}{2} + (-1)^{k+1} \cdot (k+1)^2 = \frac{(-1)^{k+1} \cdot (k+1)(k+2)}{2}.$$
   Dividing both sides by $(-1)^k \cdot (k+1)$ yields: $\displaystyle\frac{k}{2} - (k+1) = -\frac{k+2}{2}$,

   which is obviously true.

4. (a) Some motivation, like "choose 3 out of 6"; (order does not matter)
So $\binom{6}{3}$ possibilities.
$\binom{6}{3} = \frac{6!}{3!(6-3)!} = 20.$

(b) Give each boy one kind of fruit: $6 \cdot 5 \cdot 4 = 120$ possibilities
Give each girl one kind of candy: $5 \cdot 4 = 20$ possibilities.

For each allocation to the boys there are 20 possibilities for the girls, so (by the rule of product) the total number of possibilities equals: $120 \cdot 20 = 2.400$.