

Mathematics A, Solution/Correction standard, Sample Test 1

1. (a) $A_2 = [-\frac{1}{2}, 4)$, $A_4 = [-\frac{1}{4}, 8)$ [0.5 pt]

So $A_4 - A_2 = [4, 8)$. [0.5 pt]

$A_1 = [-1, 2)$, $A_2 = [-\frac{1}{2}, 4)$, \dots , $A_{10} = [-\frac{1}{10}, 20)$. [0.5 pt]

So $\bigcap_{k=1}^{10} A_k = [-\frac{1}{10}, 2)$ and $\bigcup_{k=1}^{10} A_k = [-1, 20)$. [1.5 pt]

(Argumentation: [1 pt], answers: [2 pt]).

(b) Truth table for $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$:

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	0	0
0	1	1	0	1	1	1	1
1	0	0	0	1	0	1	0
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

[2 pt]

Conclusion: The fifth and last column are not identical, so the propositions are not logically equivalent.

[1 pt]

(Correct truth table [2 pt] (column incorrect: -0.5 pt), explanation how the conclusion can be deduced from the table + correct answer [1 pt]. If table is not correct but the way the conclusion is deduced from the table is: 1 pt).

2. Basis step for $n = 1$:

$$\sum_{i=1}^1 (-1)^i \cdot i^2 = (-1)^1 \cdot 1^2 = -1 \quad \text{and also} \quad \frac{1}{2}(-1)^1 \cdot 1(1+1) = -1.$$

So the statement is correct for $n = 1$.

[0.5 pt]

Induction step:

Let $k \geq 1$ and suppose that:

$$\sum_{i=1}^k (-1)^i \cdot i^2 = \frac{1}{2}(-1)^k k(k+1) \quad (\text{Induction hypothesis: IH}).$$

[1 pt]

We must show that IH implies:

$$\sum_{i=1}^{k+1} (-1)^i \cdot i^2 = \frac{1}{2}(-1)^{k+1} (k+1)(k+2).$$

[0.5 pt]

Well:

$$\begin{aligned} \sum_{i=1}^{k+1} (-1)^i \cdot i^2 &= \sum_{i=1}^k (-1)^i \cdot i^2 + (-1)^{k+1} \cdot (k+1)^2 = \frac{1}{2}(-1)^k k(k+1) + (-1)^{k+1} \cdot (k+1)^2 \\ &= \frac{1}{2}(-1)^{k+1}(k+1)[-k + 2(k+1)] = \frac{1}{2}(-1)^{k+1}(k+1)(k+2), \end{aligned}$$

[1.5 pt]

where the second equality follows from the induction hypothesis (IH).

[0.5 pt]

(From the proof it must be crystal clear what is supposed **[1 pt]** (Induction hypothesis), what must be proved **[0.5 pt]** and in which step the induction hypothesis is used **[0.5 pt]**. In case of nonsense formulations like “Suppose it is correct FOR ALL n , so it also holds for $n + 1$ ”: at most 1 pt for the entire exercise)

3. (a) From the $20 + 30 = 50$ members, a set of 7 members must be selected, and the order in which these members are selected does not matter. So we want to know the number of 7-combinations in 50.

[0.5 pt]

The number is equal to: $\binom{50}{7}$.

[0.5 pt]

(answer: **[0.5 pt]**, (some) argumentation: **[0.5 pt]**).

- (b) The number of women must be 4, 5, 6 or 7.

The number of ways a group of 4 women can be chosen out of 30 is equal to:

$$\binom{30}{4}.$$

[0.5 pt]

For *each* choice of 4 women, there are $\binom{20}{3}$ ways to complete the committee with 3 men.

So, the number of committees with exactly 4 women is:

$$\binom{30}{4} \cdot \binom{20}{3}.$$

[1 pt]

Similarly, the number of committees with 5, 6 or 7 women respectively is equal to:

$$\binom{30}{5} \cdot \binom{20}{2}, \quad \binom{30}{6} \cdot \binom{20}{1} \quad \text{and} \quad \binom{30}{7} \cdot \binom{20}{0}.$$

[0.5 pt]

Therefore, the total number of committees that can be formed is:

$$\binom{30}{4} \cdot \binom{20}{3} + \binom{30}{5} \cdot \binom{20}{2} + \binom{30}{6} \cdot \binom{20}{1} + \binom{30}{7} \cdot \binom{20}{0}.$$

[1 pt]

(Just the answer, without any argumentation: **[1.5 pt]**).