UNIVERSITEIT TWENTE.

Tag : Toetsen/18-19/Calc1A.18-19[01].Test.EN
Course : **Calculus 1A**
Date : Friday October 26th, 2018
Time : 13:45 – 15:45

Motivate all your answers.
The use of electronic devices is not allowed.

1. The points P, Q, R and S are the vertices of a parallelogram. Three of the four vertices are given: $P(-1,-3,0)$, $Q(0,-2,4)$, and $R(-2,-1,2)$.

   (a) [1 pt] Calculate $\overrightarrow{PQ} \times \overrightarrow{PR}$.
   (b) [1 pt] Calculate the area of the parallelogram $PQRS$.
   (c) [2 pt] In the parallelogram, calculate the angle at vertex P.
   (d) [1 pt] Find an equation for the plane that contains P, Q and R.

2. [2 pt] Calculate
$$\lim_{x\to 0} \frac{x \sin x}{1-\cos x}.$$

3. Define
$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

   (a) [1 pt] Show with a calculation that $\lim\limits_{x\to 0^-} f(x) = -1$.
   (b) [2 pt] Calculate $\lim\limits_{x\to 0^+} f(x)$.
   (c) [1 pt] Is f continuous at 0? Motivate your answer.

4. (a) [1 pt] Using the definition of the derivative, show that the function $y = \sqrt[3]{x}$ is not differentiable at 0.

   Define the function $f\colon \mathbb{R} \to \mathbb{R}$ as follows:
$$f(x) = 3\sqrt[3]{x} - 4x.$$

   (b) [2 pt] Find all critical points of f.
   (c) [2 pt] Find the absolute extreme values of f on the interval $[-1,8]$.

5. [3 pt] Calculate
$$\lim_{(x,y)\to(0,0)} \frac{x+y^2}{\sqrt{x^2+y^2}},$$
or show that this limit does not exist.

6. (a) [2 pt] Find an equation for the tangent plane to the graph of the function

$$f(x, y) = x^4 - x + y^3 + 4$$

at the point $\big(1, -1, f(1, -1)\big)$. Simplify the equation as much as possible.

(b) [1 pt] Calculate the linearization of f at $(1, -1)$. With this linearization, calculate an approximation of $f\big(\frac{4}{3}, -\frac{2}{3}\big)$.

Total: 22 points