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Course : **Mathematics B1**

Practice Test

Answers

1. (a) $\mathbf{u} \times \mathbf{v} = \langle -1, 1, 2 \rangle$.

   (b) Equation for M: $x - y - 2z = 3$.

   (c) $\mathrm{proj}_{\mathbf{v}} \mathbf{u} = \langle 2, -2, 2 \rangle$.

2. (a) $\lim_{x \to 0} \dfrac{x}{\sqrt{x+1} - 1} = 2$.

   (b) The function f is continuous if

   $$2 = \lim_{x \to 0^+} \frac{x}{\sqrt{x+1} - 1} = \lim_{x \to 0^-} \frac{x}{\sqrt{x+1} - 1} = \lim_{x \to 0^-} px + q = q,$$

   hence f is continuous if $q = 2$, for all values of p.

3. Critical point on $(0, 4)$: $x_0 = \frac{1}{3}$, $f(x_0) = -\frac{2}{9}\sqrt{3}$.
   End points: $a = 0$, $f(a) = 0$ and $b = 4$, $f(b) = 6$.
   Absolute minimum is $-\frac{2}{9}\sqrt{3}$, and absolute maximum is 6.

4. (a) The limit $\lim_{(x,y) \to (0,0)} f(x, y)$ does not exist.

   Method 1: use polar coordinates: $f(x, y) = \dfrac{r^2 \cos^2 \theta - r \sin \theta}{r^2} = \cos^2 \theta - \dfrac{\sin \theta}{r}$, so $\lim_{r \to 0^+} f(x, y)$ does not exist.

   Method 2: let $x = 0$ (approach $(0, 0)$ along the y-axis), then

   $$\lim_{(x,y) \to (0,0)} f(x, y) = \lim_{y \to 0} \frac{-y}{y^2} = \lim_{y \to 0} -\frac{1}{y} = -\infty.$$

   (b) $2x - y - 2z = 1$.