

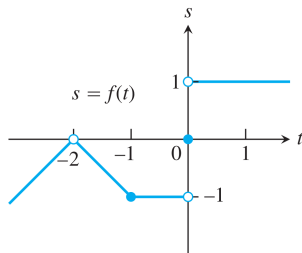
1. For the function $f(t)$ graphed below, find the following limits or explain why they do not exist:

(a) $\lim_{t \rightarrow -2} f(t)$

(b) $\lim_{t \rightarrow -1} f(t)$

(c) $\lim_{t \rightarrow 0} f(t)$

(d) $\lim_{t \rightarrow -0.5} f(t)$



Do not use the formal definition of 'limit', but rather look at section 2.2. We refer to page 66ff. for the idea.

2. Find $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x - 2}$ and $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 10}{x - 2}$.

Compare with example 1, page 65, where the numerator has been factorized.

3. (a) Graph the function $f(x) = \begin{cases} 1 - x^2 & \text{for } x \neq 1 \\ 2 & \text{for } x = 1 \end{cases}$
- (b) Find $\lim_{x \rightarrow 1^+} f(x)$ and $\lim_{x \rightarrow 1^-} f(x)$.
- (c) Does $\lim_{x \rightarrow 1} f(x)$ exist? Motivate your answer.

4. Define the function $f(x) = \begin{cases} x^2 - 1 & \text{for } -1 \leq x < 0 \\ 2x & \text{for } 0 < x < 1 \\ 1 & \text{for } x = 1 \\ -2x + 4 & \text{for } 1 < x < 2 \\ 0 & \text{for } 2 < x < 3 \end{cases}$

Graph $f(x)$ and answer the questions (a)-(f).

- (a) i. Does $f(-1)$ exist?
 ii. Does $\lim_{x \rightarrow -1^+} f(x)$ exist?
 iii. Does $\lim_{x \rightarrow -1^-} f(x) = f(-1)$?
 iv. Is f continuous at $x = -1$? *)
- (b) i. Does $f(1)$ exist?
 ii. Does $\lim_{x \rightarrow 1^+} f(x)$ exist?
 iii. Does $\lim_{x \rightarrow 1^-} f(x) = f(1)$?
 iv. Is f continuous at $x = 1$?

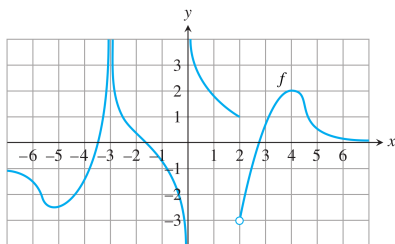
*) See p. 93 for the definition of continuity

4. (cont)

$$\text{Define the function } f(x) = \begin{cases} x^2 - 1 & \text{for } -1 \leq x < 0 \\ 2x & \text{for } 0 < x < 1 \\ 1 & \text{for } x = 1 \\ -2x + 4 & \text{for } 1 < x < 2 \\ 0 & \text{for } 2 < x < 3 \end{cases}$$

- (c) i. Is f defined at $x = 2$?
ii. Is f continuous at $x = 2$?
- (d) At what values of x is f continuous?
- (e) What value should be assigned to $f(2)$ to make the extended function continuous at $x = 2$?
- (f) To what new value should $f(1)$ be changed to remove the discontinuity at 1?

5.



Determine the following limits:

(a) $\lim_{x \rightarrow 4} f(x)$

(e) $\lim_{x \rightarrow -3^+} f(x)$

(i) $\lim_{x \rightarrow 0^-} f(x)$

(b) $\lim_{x \rightarrow 2^+} f(x)$

(f) $\lim_{x \rightarrow -3^-} f(x)$

(j) $\lim_{x \rightarrow 0} f(x)$

(c) $\lim_{x \rightarrow 2^-} f(x)$

(g) $\lim_{x \rightarrow -3} f(x)$

(k) $\lim_{x \rightarrow \infty} f(x)$

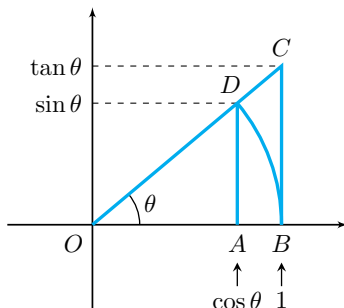
(d) $\lim_{x \rightarrow 2} f(x)$

(h) $\lim_{x \rightarrow 0^+} f(x)$

(l) $\lim_{x \rightarrow -\infty} f(x)$

6. Graph the function $y = \frac{x^3 + 1}{x^2}$ and find the asymptotes of the graph.

7.



- (a) By comparing the areas of triangle OAD , sector OBD , and triangle OBC , prove that $\sin \theta \cos \theta \leq \theta \leq \frac{\sin \theta}{\cos \theta}$ for all $0 < \theta < \frac{\pi}{2}$.
- (b) Prove that $\cos \theta \leq \frac{\sin \theta}{\theta} \leq \frac{1}{\cos \theta}$ for all $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2}) \setminus \{0\}$.
- (c) Prove that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$.