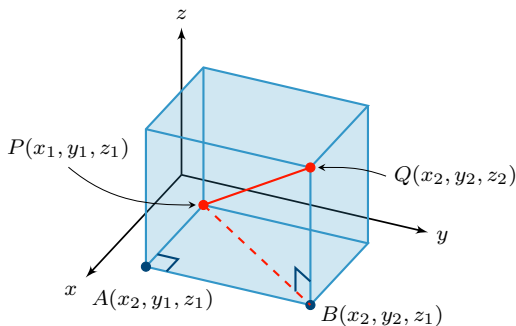


1. Determine all $\alpha \in \mathbb{R}$ such that the following vectors are orthogonal:

$$\mathbf{v} = \langle 1, 2, -3 \rangle \quad \mathbf{w} = \langle -4, \alpha, -6 \rangle.$$

2. Use the figure below to show that the distance from P to Q is equal to

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}.$$



3. Let $P(0, 4, 2)$ and $Q(1, 2, 4)$ be two points in \mathbb{R}^3 .
 - (a) Determine the line ℓ passing through P and Q .
 - (b) Determine an equation of the plane through the origin perpendicular to ℓ .
 - (c) Find equations of two planes V and W such that $V \cap W = \ell$.
4. If $\mathbf{u} = \langle -4, \alpha, -6 \rangle$ and $\mathbf{v} = \langle 1, 2, -3 \rangle$, find all $\alpha \in \mathbb{R}$ such that $|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}| |\mathbf{v}|$.
5. The parallelogram P is spanned by two vectors \mathbf{u} and \mathbf{v} .
 - (a) Describe the diagonals of P in terms of \mathbf{u} and \mathbf{v} .

A *rhombus* (Dutch: *ruit*) is a parallelogram whose edges all have the same length.
 - (b) Show that P is a rhombus if and only if the diagonals of P are orthogonal.

6. (a) Let A , B and C be points in \mathbb{R}^3 , and let $\mathbf{a} = \overrightarrow{OA}$, $\mathbf{b} = \overrightarrow{OB}$, and $\mathbf{c} = \overrightarrow{OC}$. Show that the area of the triangle ABC is equal to

$$\frac{1}{2} |(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a})|.$$

- (b) Assume that $A = (\alpha, 0, 0)$, $B = (0, \beta, 0)$ and $C = (0, 0, \gamma)$. Write the area of triangle ABC as a concise formula in α , β and γ .

