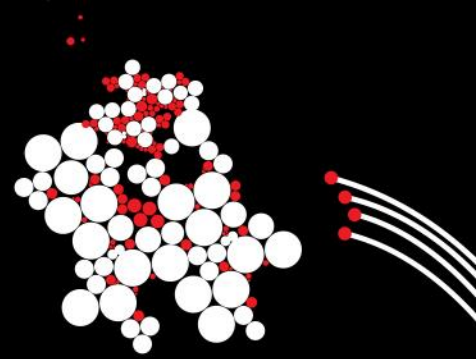


UNIVERSITY OF TWENTE.



INTRODUCTION TO MATHEMATICS LECTURE 3

JASPER DE JONG



MATHEMATICAL INDUCTION

EXAMPLE 3

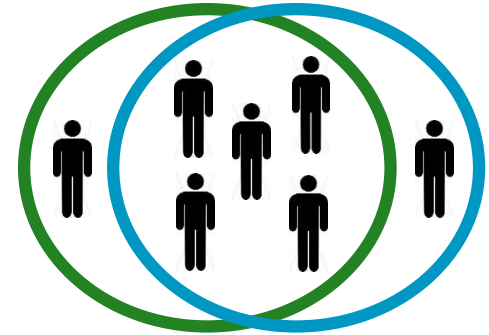
Fake Theorem

Every person has the same gender

$S(n)$: For every group of size n , all persons in that group have the same gender.

Step 1 (basis step): Each person has the same gender as him/herself.

Step 2 (induction step): Prove $\forall k \in \mathbb{N}(S(k) \rightarrow S(k + 1))$.



Proof of $\forall k \in \mathbb{N}(S(k) \rightarrow S(k + 1))$

- Suppose for some k , $S(k)$: In every group of size k , each person has the same gender. (Induction Hypothesis)
- We now need to show $S(k + 1)$: In each group $\{1, 2, \dots, k + 1\}$ of size $k + 1$ each person has the same gender.
- By the induction hypothesis, both $\{1, 2, \dots, k\}$ and $\{2, 3, \dots, k + 1\}$ have the same gender. This is the same gender for all $\{1, 2, \dots, k + 1\}$, since it is shared by $\{2, 3, \dots, k\}$.

Prepare to vote



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Voting is anonymous

Where does the proof that every person has a common gender fail?

- A. The proof does not consider the group of 0 people.
- B. Groups cannot be bigger than the number of people on earth.
- C. The induction step does not hold for all k .
- D. The basis step should not start with $n=1$.

*The question will open
when you start your
session and slideshow.*

Votes: 0



Time:
120s

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We will set these example results to zero once you've started your session and your slide show.

In the meantime, feel free to change the looks of your results (e.g. the colors).

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MATHEMATICAL INDUCTION

EXAMPLE 3

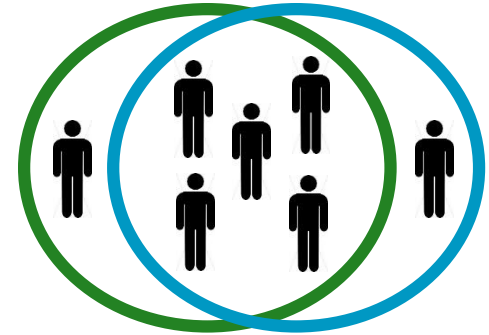
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Step 1 (basis step): Each person has the same gender as him/herself.

Step 2 (induction step): Prove $\forall k \in \mathbb{N}(S(k) \Rightarrow S(k + 1))$.



Counterexample: $S(1) \not\Rightarrow S(2)$

- Suppose $S(1)$: In every group of size 1, each person has the same gender. (Induction Hypothesis)
- We now need to show $S(2)$: In each group $\{1,2\}$ of size 2 each person has the same gender.
- By the induction hypothesis, both $\{1\}$ and $\{2\}$ have a common gender within their group. However, the groups don't have any person in common!



TODAY'S LECTURE

COMBINATORICS

- Why do we need to count (efficiently)?
 - **Not** in the supermarket.
 - Optimization
 - Probabilities

**THIS IS THE TYPE OF GUY YOU
READ ABOUT IN MATH PROBLEMS**





TODAY'S LECTURE

COMBINATORICS

- Why do we need to count (efficiently)?
 - **Not** in the supermarket.
 - **Optimization**
 - How long would it take to consider all schedules?
 - How long would a more clever method take?
 - Probabilities

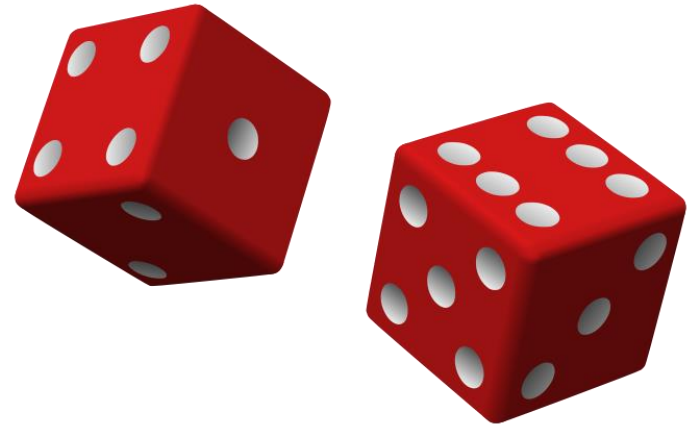




TODAY'S LECTURE

COMBINATORICS

- Why do we need to count (efficiently)?
 - **Not** in the supermarket.
 - Optimization
 - **Probabilities**



The probability that in a group of 23 people, two people have the same birthday is approximately equal to...

- A. 4%
- B. 16%
- C. 50%
- D. 91%

The question will open when you start your session and slideshow.

Votes: 0

Time: 120s

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The probability that in a group of 23 people, two people have the same birthday is approximately equal to...

- A. 4% 0.0%
- B. 16% 0.0%
- C. 50% 0.0%
- D. 91% 0.0%

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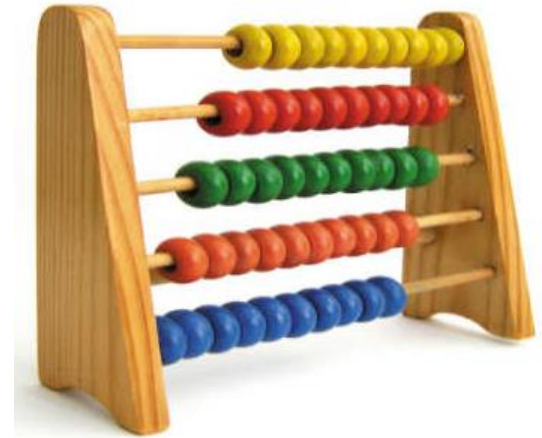
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TODAY'S LECTURE

COUNTING

- Sum/Product Rule
- Principle of Inclusion/Exclusion
- Permutations
- Combinations
 - Pascal's Triangle
 - Newton's Binomium

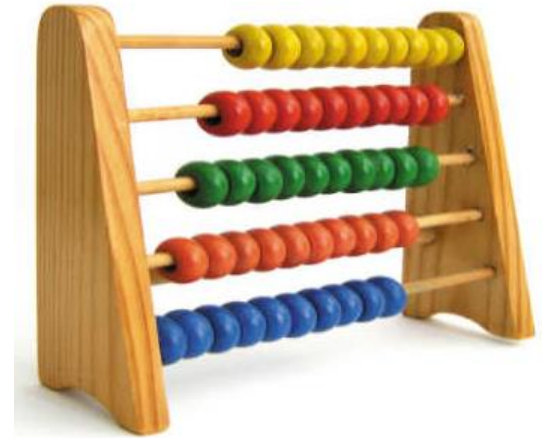




TODAY'S LECTURE

COUNTING

- **Sum/Product Rule**
- **Principle of Inclusion/Exclusion**
- Permutations
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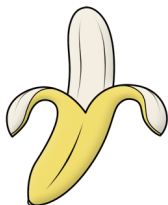
SUM RULE

Theorem (Sum rule)

If A and B are finite disjoint sets ($A \cap B = \emptyset$), then $|A \cup B| = |A| + |B|$.

Example

- I can pick my lunch by selecting from a set of 3 fruits $A = \{\text{Apple, Banana, Cherry}\}$ or a set of 2 vegetables $B = \{\text{Eggplant, Broccoli}\}$.
- Now I have $|A| + |B| = 3 + 2 = 5$ choices for lunch.



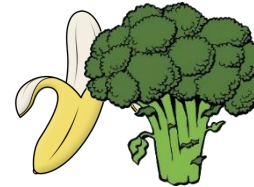
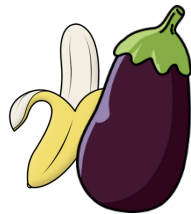
PRODUCT RULE

Theorem (Product rule)

If A and B are finite sets, then there are $|A| \cdot |B|$ different pairs (a, b) with $a \in A, b \in B$.

Example

- My lunch consists of 1 fruit and 1 vegetable from a set of 3 fruits $A = \{\text{Apple, Banana, Cherry}\}$ and a set of 2 vegetables $B = \{\text{Eggplant, Broccoli}\}$.
- Now I have $|A| \cdot |B| = 3 \cdot 2 = 6$ choices for lunch.



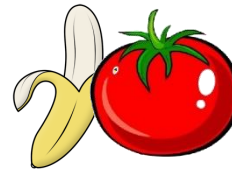
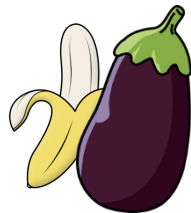
PRODUCT RULE

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If A and B are finite sets, then there are $|A| \cdot |B|$ different pairs (a, b) with $a \in A, b \in B$.

Example

- My lunch consists of 1 fruit and 1 vegetable from a set of 3 fruits $A = \{\text{Apple, Banana, Tomato}\}$ and a set of 2 vegetables $B = \{\text{Eggplant, Tomato}\}$.
- Now I have $|A| \cdot |B| = 3 \cdot 2 = 6$ choices for lunch.



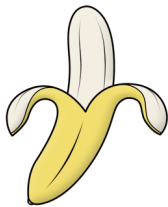
SUM RULE REVISITED

Theorem (Sum rule)

If A and B are finite **disjoint** sets ($A \cap B = \emptyset$), then $|A \cup B| = |A| + |B|$.

Example

- I can pick my lunch by selecting from a set of 3 fruits $A = \{\text{Apple, Banana, Tomato}\}$ or a set of 2 vegetables $B = \{\text{Eggplant, Tomato}\}$.
- Now I have only $4 \neq |A| + |B|$ choices for lunch.



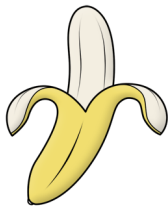
PRINCIPLE OF INCLUSION/EXCLUSION

Theorem (Principle of Inclusion/Exclusion)

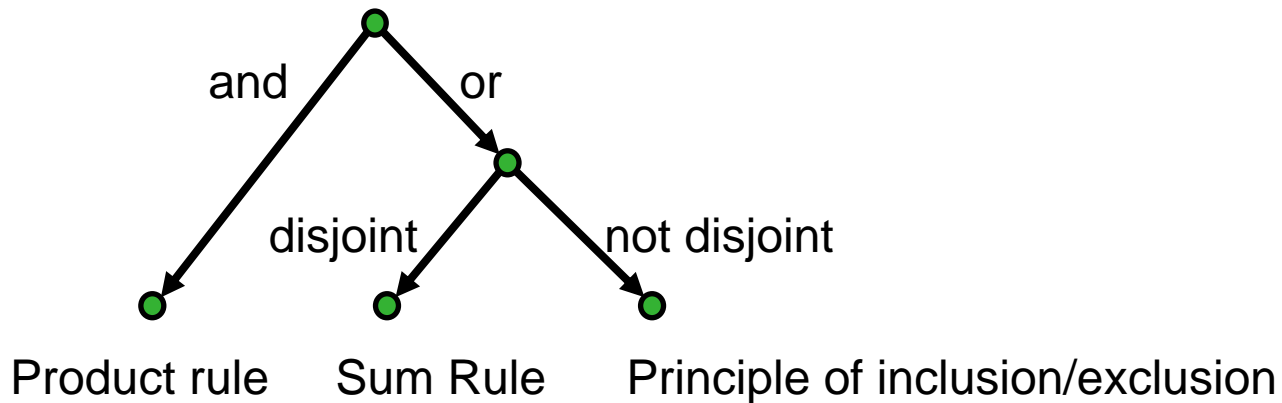
If A and B are finite sets, then $|A \cup B| = |A| + |B| - |A \cap B|$.

Example

- I can pick my lunch by selecting from a set of 3 fruits $A = \{\text{Apple, Banana, Tomato}\}$ or a set of 2 vegetables $B = \{\text{Eggplant, Tomato}\}$.
- Now I have $|A| + |B| - |A \cap B| = 3 + 2 - 1 = 4$ choices for lunch.



COMPARISON



Biologically speaking, is a tomato a fruit or a vegetable?

- A. Fruit
- B. Vegetable

The question will open when you start your session and slideshow.

Votes: 0

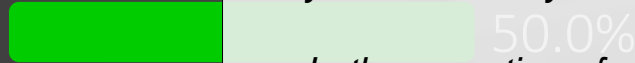


Time:
120s

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Biologically speaking, is a tomato a fruit or a vegetable?

A. Fruit



We will set these example results to zero once you've started your session and your slide show.

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B. Vegetable



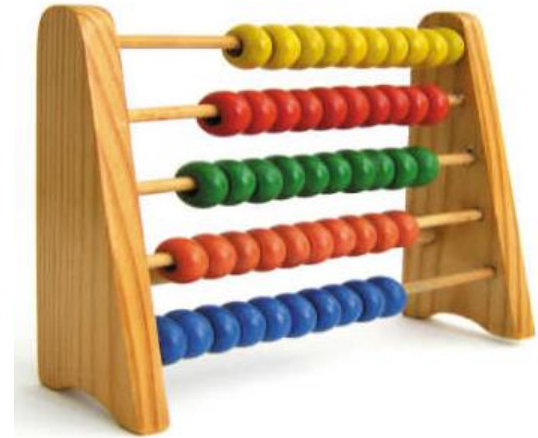
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TODAY'S LECTURE

COUNTING

- Sum/Product Rule
- Principle of Inclusion/Exclusion
- **Permutations**
- **Combinations**
 - Pascal's Triangle
 - Newton's Binomium



FOOD SELFIE

Instagram



#foodie #foodporn #nomnomnom #chefmode #cleaneating

- Goal: Making Instagram followers jealous
- Find aesthetically most pleasing way to order food

FOOD SELFIE

📷 | Instagram



#foodie #foodporn #nomnomnom #chefmode #cleaneating

- Goal: Making Instagram followers jealous
- Find aesthetically most pleasing way to order food

How many possible ways exist to orders 6 foods for a food selfie? For example: (Cherry, Tomato, Banana, Apple, Broccoli, Eggplant)

- A. 15
- B. 36
- C. 720
- D. 46656

*The question will open
when you start your
session and slideshow.*

Votes: 0 ● Time:
120s

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We will set these example results to zero once you've started your session and your slide show.

B. 36



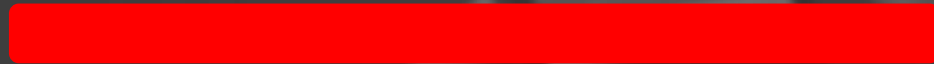
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C. 720



75.0%

D. 46656



100.0%

Close
d



PERMUTATIONS

Solution

- In how many ways can you order six distinct objects?
- To order means to assign each position to an object.
- Six possible objects for first position.
- Then, five remaining objects for second position.
- Etc...
- Finally, one object for sixth position.
- By product rule: $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$ possible orderings.

Definition Permutation

A permutation of n distinct elements is an ordering of the n elements.

There are $n! = n \cdot (n - 1) \cdot \dots \cdot 2 \cdot 1$ possible permutations of n elements.

r -PERMUTATIONS

Example

- Suppose we want to order four out of six possible foods {apple, banana, cherry, broccoli, eggplant and a tomato}.
- For example: (tomato, apple, eggplant, broccoli)
- Number of possible orderings:

$$6 \cdot 5 \cdot 4 \cdot 3 = \frac{\overbrace{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}^n}{\underbrace{2 \cdot 1}_{n-r}} = \frac{6!}{2!}$$

Definition r -Permutation

An r -permutation of n distinct elements is an ordering of r out of n elements, where $0 \leq r \leq n$

There are $P(n, r) = n \cdot (n - 1) \cdot \dots \cdot (n - r + 1) = \frac{n!}{(n-r)!}$ possible r -permutations of n elements

Instead of ordering the fruits for a food selfie, I want to select 2 out of 4 fruits to put into my fruit salad. How many possible combinations of 2 fruits are there?

- A. 6
- B. 12
- C. 18
- D. 24

The question will open when you start your session and slideshow.

Votes: 0

Time: 120s

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Instead of ordering the fruits for a food selfie, I want to select 2 out of 4 fruits to put into my fruit salad. How many possible combinations of 2 fruits are there?

A. 6



25.0%

B. 12



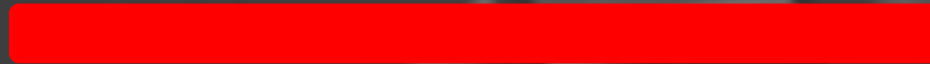
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C. 18



75.0%

D. 24



100.0%

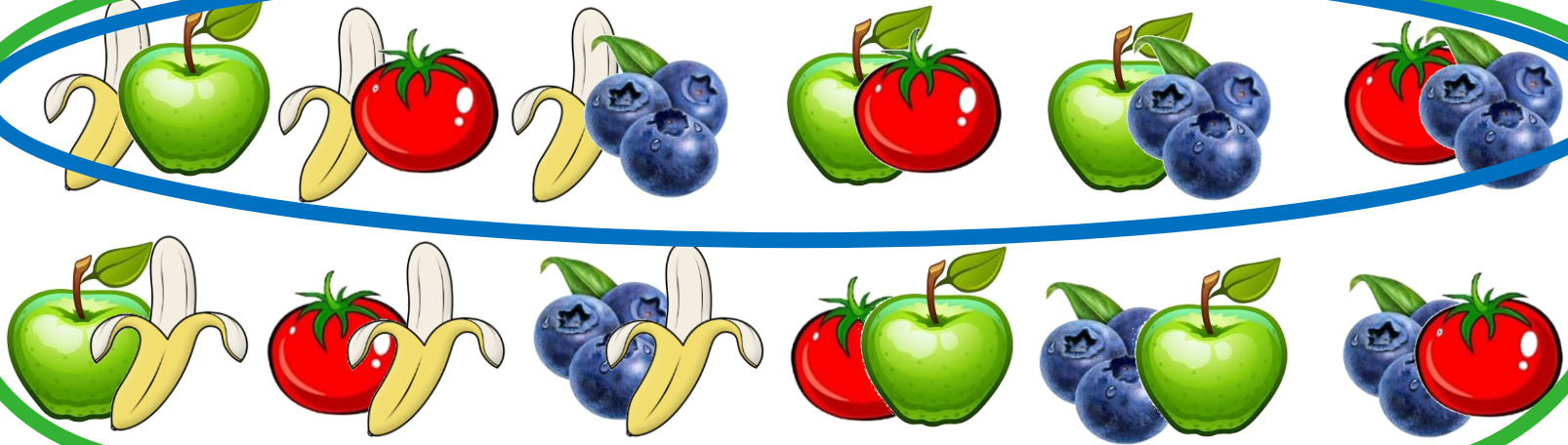
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Close
d

SOLUTION

2-Combinations



2-Permutations



COMBINATIONS

Definition Combination

An r -combination of n distinct elements is an unordered subset of r out of n elements, where $0 \leq r \leq n$

There are $C(n, r) = \binom{n}{r} = \frac{P(n, r)}{r!} = \frac{n!}{r! \cdot (n-r)!}$ possible r -combinations of n elements.

COMPARISON

Permutations	Combinations
$P(n, r)$	$C(n, r) = \binom{n}{r}$
Order matters	Order doesn't matter
$\frac{n!}{(n-r)!}$	$\frac{n!}{r! \cdot (n-r)!}$



BIRTHDAY PARADOX

Theorem (Birthday Paradox)

In a group of 23 people, the probability that at least 2 share their birthday is approximately 50%

Solution

- A scenario is an allocation of birthdays to the 23 people, e.g. (March 14th, January 5th, ...)
- For simplicity assume that all scenario's are equally likely.
- Total number of scenario's: 365^{23} (product rule)
- Number of scenario's for which no pair shares a birthday: $365 \cdot 364 \cdot \dots \cdot (365 - 23 + 1) = \frac{365!}{342!}$
- Number of scenario's for which at least one pair shares a birthday: $365^{23} - \frac{365!}{342!}$
- Probability at least one pair shares birthday: $\frac{365^{23} - \frac{365!}{342!}}{365^{23}} \approx 50\%$

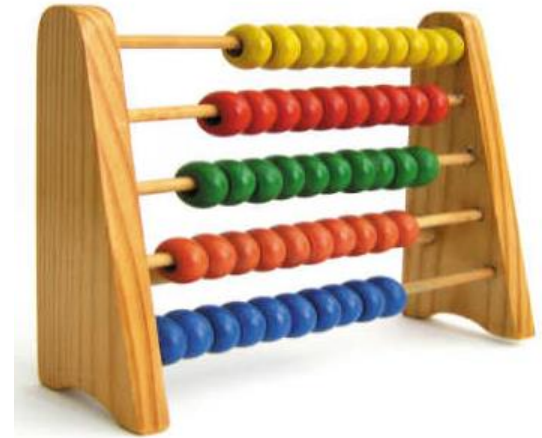
For the test, you **don't** need to know probabilities, but you can compute the number of scenario's.



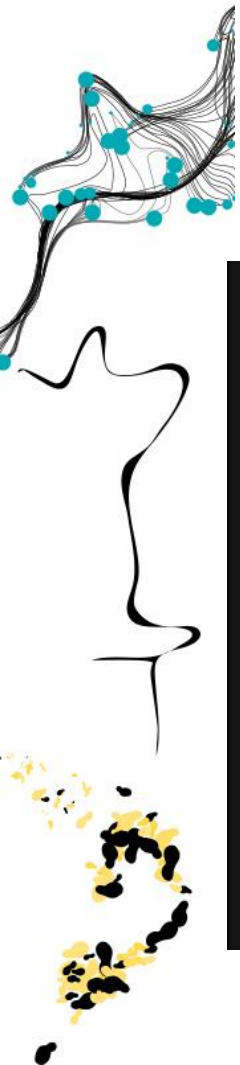
TODAY'S LECTURE



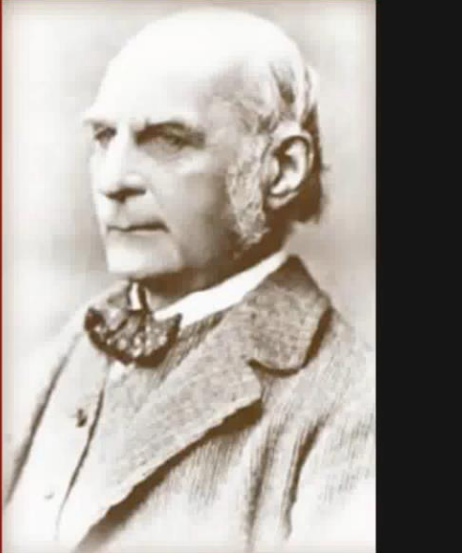
COUNTING

- Sum/Product Rule
- Principle of Inclusion/Exclusion
- Permutations
- Combinations
 - **Pascal's Triangle**
 - **Newton's Binomium**



GALTON BOARD



 <p>Technical University of Crete Dept. of Production Engineering & Management University Campus - Kounoupidiana, 73100 Chania GREECE</p>	<p>www.m3.tuc.gr</p>
 <p>Micromachining & Manufacturing Modeling Lab Building E4.026 - Tel. +30 28210 37484 - m3@dpem.tuc.gr</p>	
<p>■ ■ ■ Galton's board</p> <p>Kasiotakis Vasilis Crete 2012</p>	

Why do the balls that fall through the Galton board distribute unevenly over the different receptacles?

Choose the best answer.

- A. Just Coincidence.
- B. Gravity.
- C. Paths near the sides of the board are longer.
- D. There are more paths to receptacles in the middle.

The question will open when you start your session and slideshow.

Votes: 0



Time:
120s

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Why do the balls that fall through the Galton board distribute unevenly over the different receptacles? Choose the best answer.

A. Just Coincidence.

B. Gravity.


C. Paths near the sides of the board are longer.

D. There are more paths to receptacles in the middle.

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 100.0%

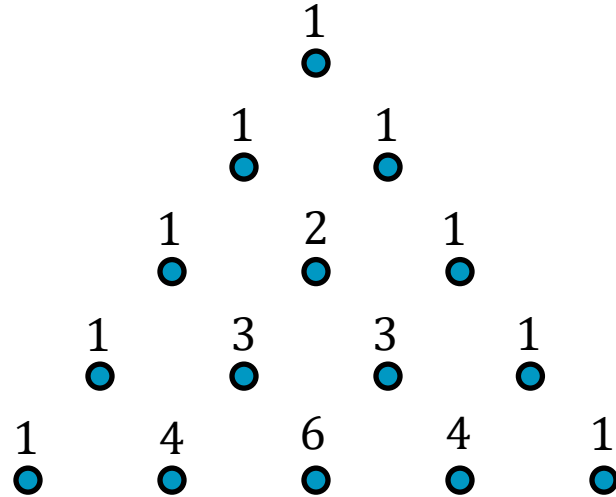
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PASCAL'S TRIANGLE

SOLUTION

Numbers denote number of ways to arrive at a position.



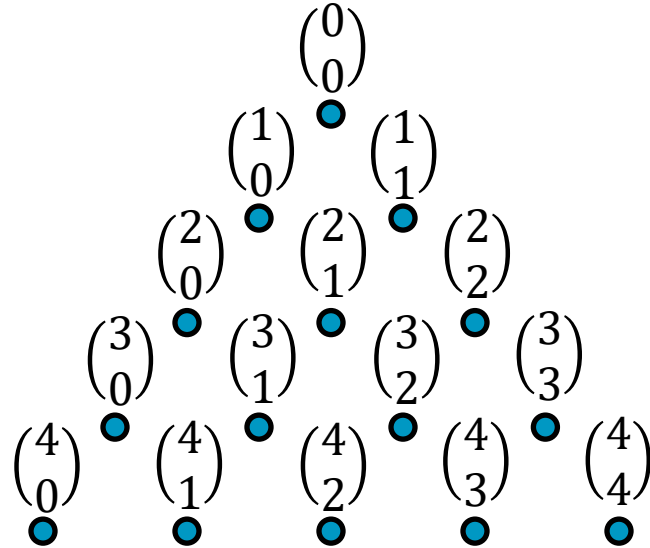


PASCAL'S TRIANGLE

SOLUTION

Numbers denote number of ways to arrive at a position.

$\binom{n}{r}$: going right r out of n times.





PASCAL'S TRIANGLE

NEWTON'S BINOMIAL THEOREM

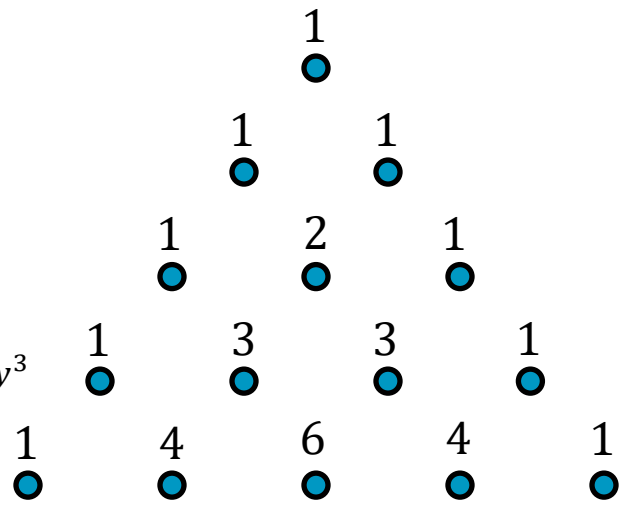
Numbers denote number of ways to arrive at a position.

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

Binomial Theorem

$$(x + y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$$





PASCAL'S TRIANGLE

NEWTON'S BINOMIAL THEOREM

$$(x + y)^2 = (x + y) \cdot (x + y) = x \cdot x + x \cdot y + y \cdot x + y \cdot y = x^2 + 2xy + y^2$$

Choose the left term from both factors

Choose the left term once and the right term once

$$(x + y)^3 = x \cdot x \cdot x + x \cdot x \cdot y + x \cdot y \cdot x + x \cdot y \cdot y + y \cdot x \cdot x + y \cdot x \cdot y + y \cdot y \cdot x + y \cdot y \cdot y$$
$$= x^3 + 3x^2y + 3xy^2 + y^3$$

Choose the left term from all three factors

Choose the left term once and the right term twice

Binomial Theorem

$$(x + y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$$



SUMMARY

- And: Product, Or: Sum if disjoint, otherwise principle of inclusion/exclusion.
- Permutations: Order matters.
- Combinations: Order does not matter.

Binomial Theorem

$$(x + y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$$