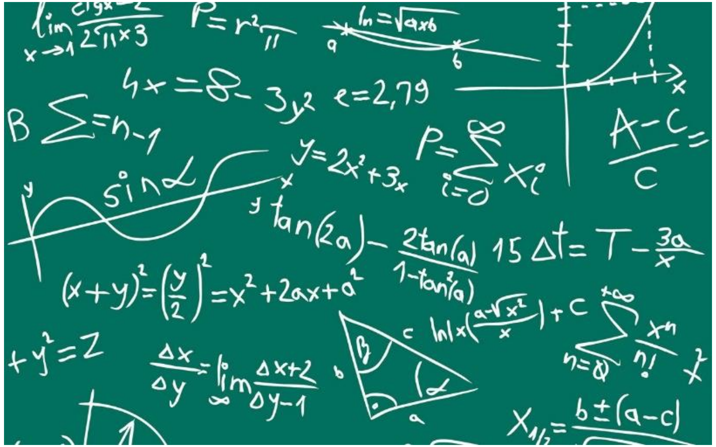
UNIVERSITY OF TWENTE.

# INTRODUCTION TO MATHEMATICS

JASPER DE JONG

# "IS MATH RELATED TO SCIENCE?"

"Is Math related to Science?"



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Voting is anonymous

## Who said: "Is Math related to Science?" ?

A. Justin Bieber
B. Kanye West
C. Katy Perry
D. Kim Kardashian

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# Who said: "Is Math related to Science?" ?

| | | |
|---|---|---|
| A. | Justin Bieber | 0.0% |
| B. | Kanye West | 0.0% |
| C. | Katy Perry | 0.0% |
| D. | Kim Kardashian | 0.0% |

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# "IS MATH RELATED TO SCIENCE"

# “IS MATH RELATED TO SCIENCE?”

“Mathematics is the language in which God has written the universe.”

*-Galileo Galilei*

Goal Introduction to Mathematics

- How to ‘speak’ math…
- …and a bit of combinatorics. (3rd week)

# WHY MATH?
## WHAT IS WRONG WITH ENGLISH?

Formal language prevents confusion.

**Example**

Three logicians walk into a bar.
Waitress: ”Would you all like a beer?”

Logician 1: “I don’t know”.

Logician 2: “I don’t know”.

Logician 3: “Yes”.

How many beers should the waitress serve?

# How many beers should the waitress serve?

A. 0
B. 1
C. 2
D. 3

Waitress: ”Would you all like a beer?”
Logician 1: “I don’t know”.
Logician 2: “I don’t know”.
Logician 3: “Yes”.

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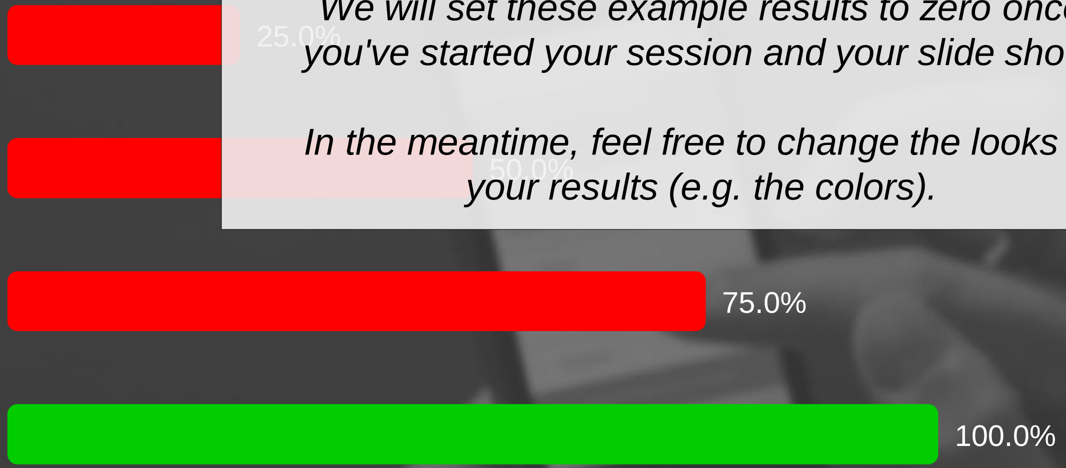
# How many beers should the waitress serve?

A. 0

B. 1

C. 2

D. 3

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# SOLUTION

Formal language prevents confusion.

**Example**

Three logicians walk into a bar.
Waitress: ”Would you all like a beer?”

Logician 1: “I don’t know”.

Logician 2: “I don’t know”.

Logician 3: “Yes”.

How many beers should the waitress serve?

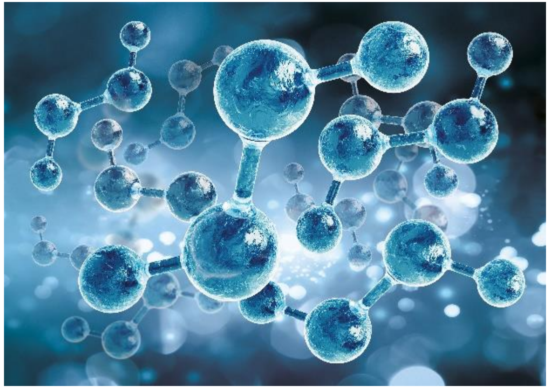
**Solution**

- Logician 1 wants a beer, otherwise he would have answered “No”.
- Logician 2 wants a beer, otherwise he would have answered “No”.
- Logician 3 draws the same conclusion. Since he wants a beer himself, his answer is “Yes”.

# WHY MATH?
## WHAT IS WRONG WITH ENGLISH?

Formal language prevents confusion.

- ***Not*** with friends
- ***Not*** in familiar situations
- ***It does*** in science:
  - “The experiment indicates that not all molecules have property x”
  - “The experiment indicates that all molecules do not have property x”



# TODAY'S LECTURE
## CONTENTS

- Organization Introduction to Mathematics
- Sets
- Logic

# TODAY'S LECTURE
## CONTENTS

- **Organization Introduction to Mathematics**
- Sets
- Logic

# ORGANIZATION
## STUDY MATERIAL

- Union Shop: Reader: nr 510; Introduction to Mathematics August 2018, € 7,00.
- Course Information: canvas.utwente.nl, enroll for the 'course': https://canvas.utwente.nl/enroll/9T4TKG or search for 'math' in 'courses -> all courses -> browse more courses'.
- After enrolling, you can simply access the course under 'courses'

# ORGANIZATION
## LEARNING FORMATS

- Lectures
  - Big lines, not all material covered. Always on Monday morning.
- Guided selfstudies
  - Evaluate the theory of the lecture and make preliminary exercises under guidance of a teacher.
- Tutorials
  - Make the exercises in the course schedule. Teacher available for questions.
- MyLabsPlus
  - On several Mondays, before the lecture, there is a digital test.
    See Canvas for more information about MyLabsPlus.
- (Unguided) selfstudies
  - Make unfinished exercises of the tutorial. Questions can be asked in the next tutorial.

# ORGANIZATION
## BEHAVIOURAL RULES/GUIDELINES

- Lectures
  - Be on time!
  - Laptops remain closed. Not for yourself, but for your fellow students!
- (Guided) self-studies, tutorials, MyLabsPlus
  - Like any language, the easiest way to learn is by ***doing***.
  - Students who do attend, obtain higher grades on average.
  - Prepare questions!

# TODAY'S LECTURE
## CONTENTS

- Organization Introduction to Mathematics
- **Sets**
- Logic

# SETS
## DEFINITION

**Definition Set**

A ***set*** is a well-defined unordered collection of distinct elements

**Examples**

- $\{1,3,6\}$
- {Jan, Pier, Tjoris, Corneel}
- $\{\{1,3,6\},$ {Jan, Pier, Tjoris, Corneel}$, 2\}$
- $\{1,2,3,\ldots\}$

- Unordered: $\{1,3,6\} = \{1,6,3\}$
- Distinct elements: $\{1,3,6\} = \{1,6,3,6\}$
- Well-defined (not ambiguous): ′All friendly people′ is not a set.

# SETS

## NOTATION

**Definition Set**

A ***set*** is a well-defined unordered collection of distinct elements

- Always use curly brackets!
- Within the curly backets describe:
  - Option 1: All elements
  - Option 2: Pattern
  - Option 3: Properties

**Example**

$\{1,2,3,4,5,6,7,8,9,10\} = \{1,2,3,\ldots,10\} = \{n \mid n$ is an integer with $1 \leq n \leq 10\}$

- The |-symbol means “for which”.
- The ∈-symbol means "Is an element of“.

**Example**

$4 \in \{1,2,3,\ldots,10\}$

# SETS
## SUBSETS

**Definition Subset $A \subseteq B$**

Every element $a \in A$ is also an element of $B$ (if $a \in A$, then $a \in B$)

**Definition Proper Subset $A \subset B$**

$A \subseteq B$ and $A \neq B$

**Definition Equal Sets $A = B$**

$A \subseteq B$ and $B \subseteq A$ ($A$ and $B$ contain exactly the same elements)

**Examples**

- $\{1,5\} \subseteq \{1,2,3,\ldots\}$
- $\{1,5\} \subset \{1,2,3,\ldots\}$
- $\{1,5\} \subseteq \{1,5\}$
- $\{1,5\} \not\subset \{1,5\}$
- $\{1,5\} \nsubseteq \{\{1,5\},\{2,5\},1\}$
- $\{1,5\} \in \{\{1,5\},\{2,5\},1\}$

# SETS
## EMPTY SET

**Definition Empty set $\emptyset$**

$\emptyset$ is a set without any element ($x \notin \emptyset$ for all $x$)

How many different empty sets exist?

**Definition Empty set ∅**

∅ is a set without any element ($x \notin \emptyset$ for all $x$)

A. Zero, if the empty set would exist, then ∅∈{∅}, so it is not empty.

B. One, all empty sets contain exactly the same elements: no elements.

C. Two: ∅ and {}.

D. Infinitely many: all sets {x with P(x)}, for which P(x) is false.
For example P(x):x≠x or P(x):x>x.

*The question will open when you start your session and slideshow.*

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# How many different empty sets exist?

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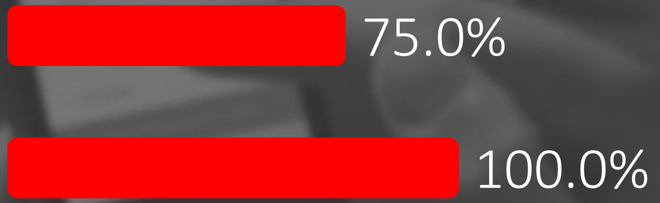
*In the meantime, feel free to change the looks of your results (e.g. the colors).*

A. Zero, if the empty set would exist, then ∅∈{∅}, so it is not empty.

B. One, all empty sets contain exactly the same elements: no elements.

C. Two: ∅ and {}.

D. Infinitely many: all sets {x with P(x)}, for which P(x) is false. For example P(x):x≠x or P(x):x>x.

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# SETS

## EMPTY SET

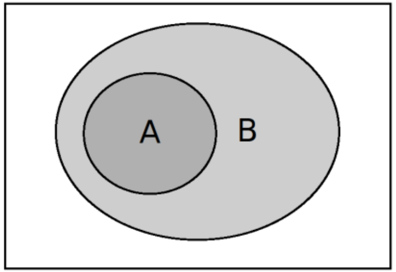
**Definition Empty set $\emptyset$**

$\emptyset$ is a set without any element ($x \notin \emptyset$ for all $x$)

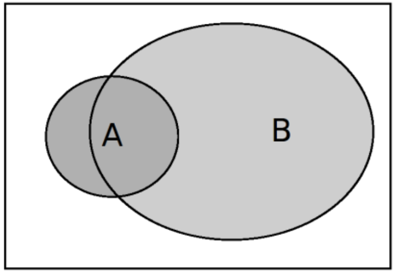
**Solution**

- One set can be described in multiple ways, so the arguments for two and infinitely many are wrong.
- $\emptyset \neq \{\emptyset\}$. $\emptyset = \{\}$. $\{\emptyset\}$ does contain an element: $\emptyset$. The argument for zero is also wrong.
- Suppose that there are multiple empty sets, we denote two of them by $\emptyset$ en $\emptyset'$.
  $\emptyset$ contains no elements, so $\emptyset \subseteq \emptyset'$. $\emptyset'$ contains no elements, so $\emptyset' \subseteq \emptyset$.
  Therefore $\emptyset' = \emptyset$. There exists only one empty set.

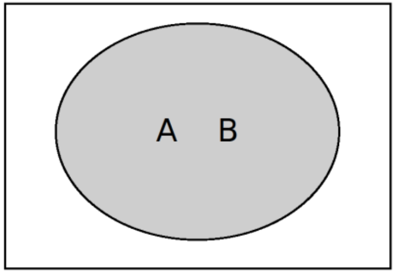
# SETS
## VENN-DIAGRAMS




$A \subset B$




$A \not\subseteq B$ en $B \not\subseteq A$



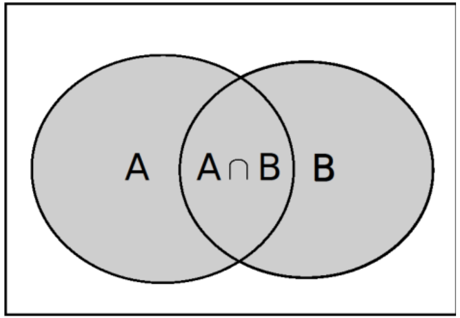

$A = B$

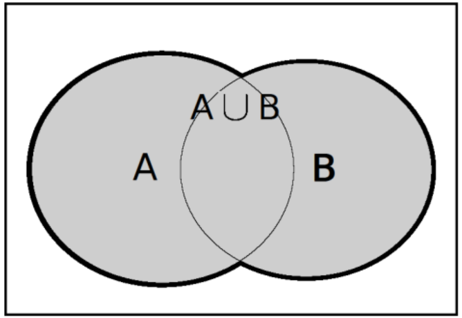
# SETS
## INTERSECTION AND UNION

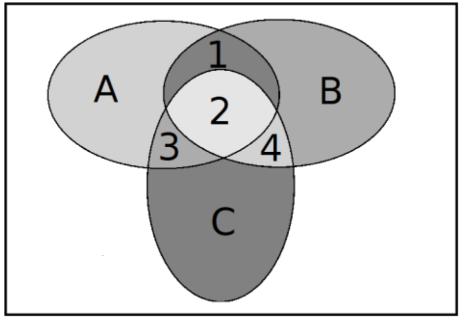
**Definitions Intersection ∩ and Union ∪**

$A \cap B = \{x | x \in A \text{ and } x \in B\}$

$A \cup B = \{x | x \in A \text{ or } x \in B\}$




$A \cap B \cap C$: area 2

Given sets A and B with:

$(A \cap B) \subseteq B$

$(A \cup B) \subseteq A$.

Which of the following propositions is always true?

A. $A \subseteq B$

B. $B \subseteq A$

C. $B = A$

D. $B = \emptyset$

**Definitions Intersection ∩ and Union ∪**

$A \cap B = \{x | x \in A$ en $x \in B\}$

$A \cup B = \{x | x \in A$ of $x \in B\}$

*The question will open when you start your session and slideshow.*

Time: 120s

Given sets A and B with:

(A∩B) ⊆ B

(A∪B) ⊆ A.

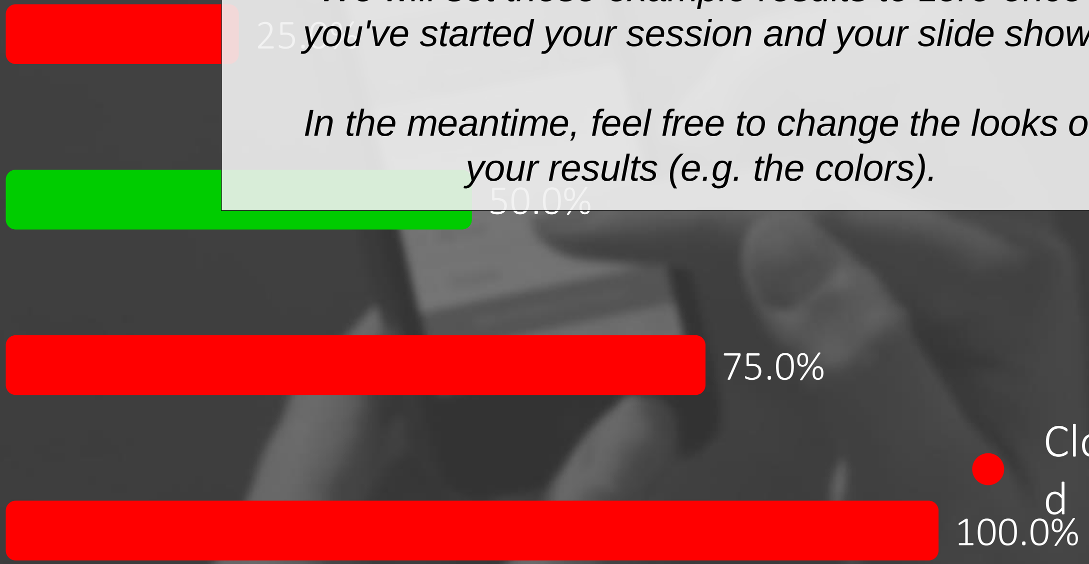
Which of the following propositions is always true?

A. A ⊆ B

B. B ⊆ A

C. B = A

D. B = Ø

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# SETS

## INTERSECTION AND UNION

**Definitions Intersection ∩ and Union ∪**

$A \cap B = \{x | x \in A$ and $x \in B\}$

$A \cup B = \{x | x \in A$ or $x \in B\}$

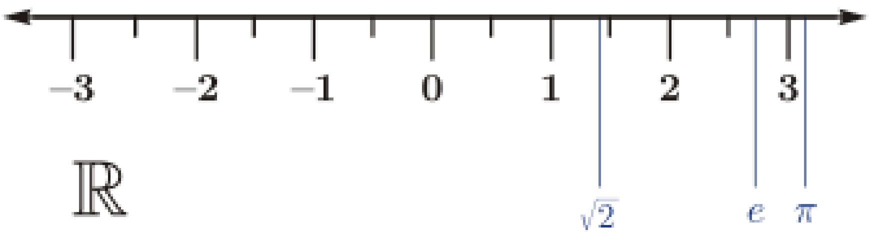
**Solution**

- Given:
  - $(A \cap B) \subseteq B$
  - $(A \cup B) \subseteq A.$
- From $(A \cap B) \subseteq B$ we can't conclude anything, since $A \cap B$ is always a subset of $B.$
- $(A \cup B) \subseteq A$ means that every element of $A \cup B$ is also in $A$. In particular, $A \cup B$ contains all elements of $B$, so every element of $B$ is also in $A$. Therefore $B \subseteq A$.

# SETS
## IMPORTANT SETS

**Definitions $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$**

- Natural numbers $\mathbb{N} = \{1,2,\ldots\}$
- Integers $\mathbb{Z} = \{0,1,-1,2,-2,\ldots\}$
- Rational Numbers $\mathbb{Q} = \left\{\frac{a}{b} \middle| a, b \in \mathbb{Z}, b \neq 0\right\}$
- Real numbers $\mathbb{R}$ 'All numbers on the real number line'




**Observation:** $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subseteq \mathbb{R}$

Also $\mathbb{Q} \subset \mathbb{R}$?

Answer next week!

# TODAY'S LECTURE

## CONTENT

- Organization Introduction to Mathematics
- Sets
- **Logic**

# LOGIC

## PROPOSITIONS

**Example**

**Propositions** “It rains.” and “If it rains, I will stay inside.”
**Conclusion** “I will stay inside.”

**Example**

**Propositions** “If it rains, I will stay inside.” and “I will stay inside.”
**Conclusion** ???

**Definition Proposition**

A proposition is a statement whose truth can be expressed by the values ‘true’ and ‘false’.

# LOGIC

## PROPOSITIONS

**Definition Proposition**

A proposition is a statement whose truth can be expressed by the values 'true' and 'false'.

**Examples Propositions**

- $1 + 3 > 5$.
- 2 is an even integer.
- If it rains, I will stay inside.
- For all natural numbers $n \in \mathbb{N}$ (it holds that) $n \geq 0$.
- All cars are red.
- There exist red cars.

**Examples non-Propositions**

- $1 + n > 5$.
- If it rains, person $x$ stays inside.
- FC Twente is a good soccer club.

# LOGIC

## OPERATORS

| Symbol | Name | Proposition… | … is true if and only if |
|---|---|---|---|
| $\wedge$ | and-operator | $p \wedge q$ | $p$ is true and $q$ is true. |
| $\vee$ | or-operator | $p \vee q$ | $p$ is true and/or $q$ is true. |
| $\neg$ | not-operator | $\neg p$ | $p$ is false. |
| $\rightarrow$ | implication-operator | $p \rightarrow q$ | if $p$ is true, then $q$ is true. |
| $\leftrightarrow$ | equivalence-operator | $p \leftrightarrow q$ | $(p \rightarrow q) \wedge (q \rightarrow p)$. |

### Examples

| | | |
|---|---|---|
| $p$ | It rains. | (given) |
| $q$ | I will stay inside. | (given) |
| $p \wedge q$ | It rains and I will stay inside. | |
| $p \rightarrow q$ | If it rains, then I will stay inside. | |
| $\neg p \rightarrow q$ | If it does not rain, then I will stay inside. | |
| $\neg(p \rightarrow q)$ | It is not true that I will stay inside if it rains. | |

# LOGIC
## OPERATORS

| Symbol | Name | Proposition… | … (is true) if and only if |
|---|---|---|---|
| $\wedge$ | and-operator | $p \wedge q$ | $p$ (is true) and $q$ (is true). |
| $\vee$ | or-operator | $p \vee q$ | $p$ (is true) and/or $q$ (is true). |
| $\neg$ | not-operator | $\neg p$ | $p$ is false. |
| $\rightarrow$ | implication-operator | $p \rightarrow q$ | if $p$ (is true), then $q$ (is true). |
| $\leftrightarrow$ | equivalence-operator | $p \leftrightarrow q$ | $(p \rightarrow q) \wedge (q \rightarrow p)$. |

**Examples**

- $p$ It rains. (given)
- $q$ I will stay inside. (given)
- $p \wedge q$ It rains and I will stay inside.
- $p \rightarrow q$ If it rains, then I will stay inside.
- $\neg p \rightarrow q$ If it does not rain, then I will stay inside.
- $\neg(p \rightarrow q)$ It is not true that I will stay inside if it rains.

# LOGIC

## TRUTH TABLES

| Symbol | Name | Proposition… | … if and only if |
|---|---|---|---|
| $\wedge$ | and-operator | $p \wedge q$ | $p$ and $q$. |
| $\vee$ | or-operator | $p \vee q$ | $p$ and/or $q$. |
| $\neg$ | not-operator | $\neg p$ | $p$ is false. |
| $\rightarrow$ | implication-operator | $p \rightarrow q$ | if $p$, then $q$. |
| $\leftrightarrow$ | equivalence-operator | $p \leftrightarrow q$ | $(p \rightarrow q) \wedge (q \rightarrow p)$. |

1 means ‘true’. 0 means ‘false’.

| $p$ | $q$ | $\neg p$ | $p \wedge q$ | $p \vee q$ | $p \rightarrow q$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |

Which of the following propositions are true?

If 1+1=3, then Enschede is a Dutch city.

If Enschede is a German City, I will eat my hat.

A. Both propositions are false.
B. Proposition 1 is true, propositie 2 is false.
C. Proposition 1 is false, proposition 2 is true.
D. Both propositions are true.

*The question will open when you start your session and slideshow.*

Time:
120s

Votes: 0

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A. Both propositions are false.

B. Proposition 1 is true, propositie 2 is false.

C. Proposition 1 is false, proposition 2 is true.

D. Both propositions are true.

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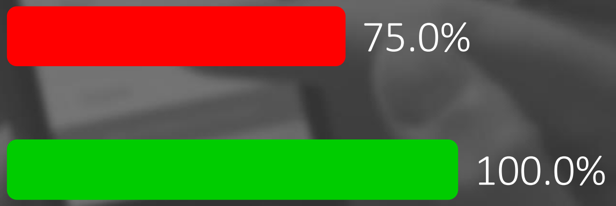


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# LOGIC

## TRUTH TABLES

### Solution

- 'If $p$, then Enschede is a Dutch city' is true regardless of proposition $p$: Enschede is a Dutch city.
- 'If $p$, then I will eat my hat' you can safely say, as long as $p$ is false. In this case 'Enschede is a German city' is false, so the whole proposition is true.
- Equivalent: 'Either Enschede is not a German city, or I will eat my hat.'

| $p$ | $q$ | $p \rightarrow q$ | $\neg p \vee q$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 |

# LOGIC

## PREDICATES

### Examples Predicates

- $1 + n > 5$.
- If it rains, person $x$ will stay inside.
- $n^2 < 0$.

### Definition Predicate

A predicate is a statement that can contain variables that influence the truth value of the statement.

Predicates can turn into propositions:

- Choose a value.
- Statement about all values.
- Statement about the existence of a value.

### Examples Propositions

- $1 + 8 > 5$.
- If it rains, all persons stay inside.
- There exists an integer $n \in \mathbb{Z}$ such that $n^2 < 0$.

# LOGIC
## QUANTIFIERS

### Notation Quantifiers

- ∀ universal quantifier 'For all'
- ∃ existential quantifier 'There exists a'

### Examples Propositions

| | | |
|---|---|---|
| $p$ | It rains. | (given) |
| $q(x)$ | Person $x$ will stay inside. | (given) |
| ▪ $p \to (\forall x\, q(x))$ | If it rains, all persons stay inside. | |
| ▪ $\exists n \in \mathbb{Z}\ (n^2 < 0)$. | There exists an integer $n$ such that $n^2 < 0$. | |

# TO CONCLUDE…

- Learn the mathematical language:
  - Attend guided self-studies and tutorials.
  - Ask questions.

- Next week:
  - Formal proofs…
    …using this week's notation!