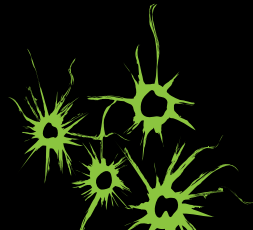
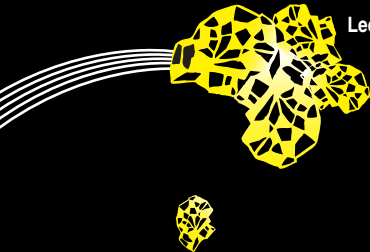


Model-Based Testing with Labeled Transition Systems

Software Testing and Risk Assessment

Lecture 4b





Today

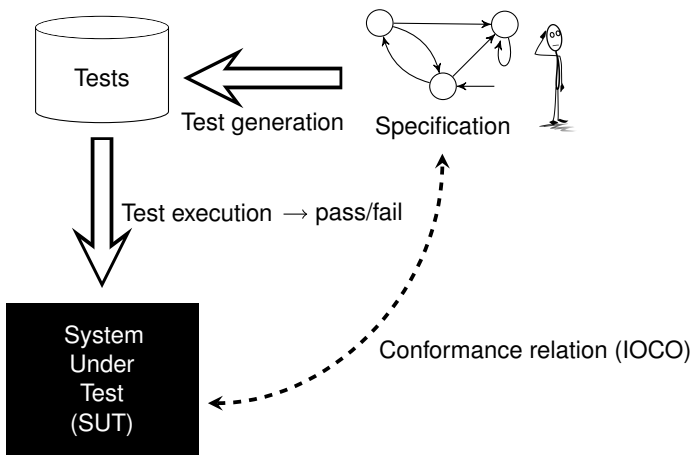
- ▶ Project planning
- ▶ Model-based testing
- ▶ Labeled Transitions Systems
- ▶ Quiescence
- ▶ Nondeterministic LTS
- ▶ Formal definitions
- ▶ Determinisation
- ▶ Test cases
- ▶ Distinguishing and homing test cases



Plan tasks for Software Testing Project

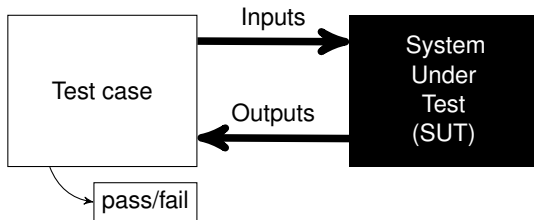
- ▶ Last homework on March 26
- ▶ SmartDoor assignment on March 31
- ▶ Axini lectures on March 24 and April 1
- ▶ Project deadline on April 11

Model-Based Testing



Model-based testing in this course

- ▶ What kind of testing?
 - ▶ Functionality: functional behaviour and interaction of system
 - ▶ On system-level
 - ▶ Black-box: use only inputs and outputs
 - ▶ Test execution:



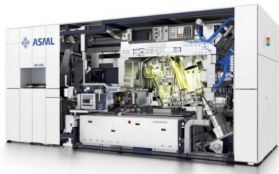


Model-based testing in this course

- ▶ What kind of systems?
 - ▶ Systems with a software interface
 - ▶ Discrete event-driven systems
 - ▶ Reactive, dynamic systems
 - ▶ Data-intensive systems

Model-based testing in this course

- ▶ What kind of systems?
 - ▶ Systems with a software interface
 - ▶ Discrete event-driven systems
 - ▶ Reactive, dynamic systems
 - ▶ Data-intensive systems
- ▶ Real-world systems:



ASML wafer machine



Canon industrial printer



Model-based testing in this course

- ▶ What kind of model?
 - ▶ Lecture 4b & 5: Labeled Transition Systems (LTS)
 - ▶ Lecture 6: Symbolic Transition System (STS)
 - ▶ Lectures 7: AML model (Axini)



Model-based testing in this course

- ▶ What kind of model?
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 - ▶ What vs. How



Model-based testing in this course

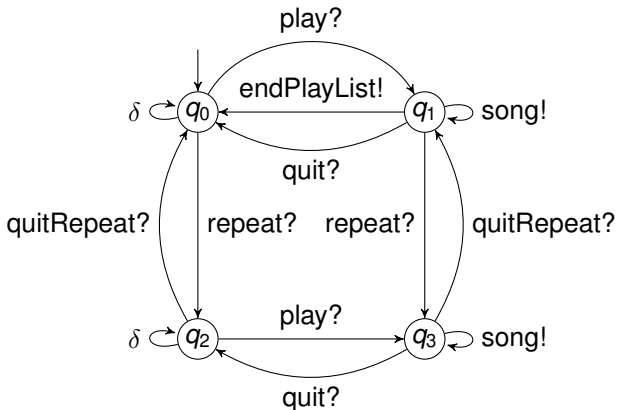
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 - ▶ Smaller size \implies easier to maintain
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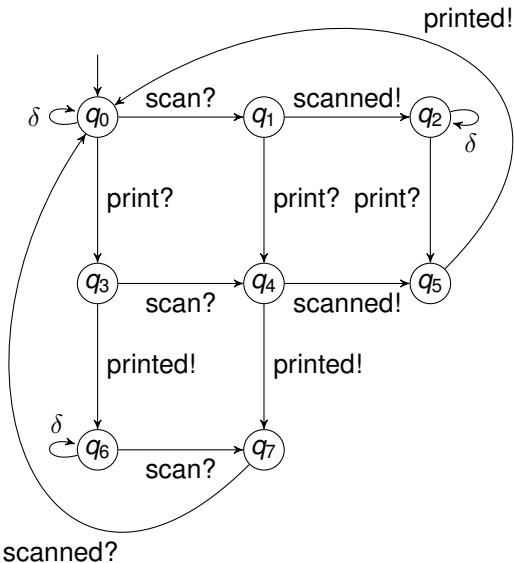
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 - ▶ Smaller size \implies easier to maintain
 - ▶ Same size \implies Test larger systems
- ▶ Test models may be partial
 - ▶ Only model relevant/most important part

Example LTS: Music App

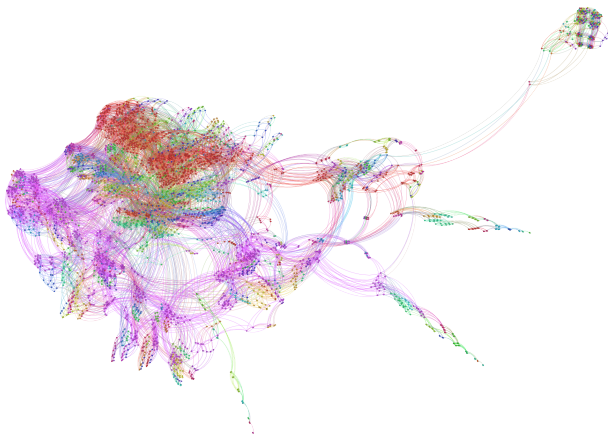


Example LTS: printer/scanner



Size of a model of a real-world system¹

- ▶ A model of a Canon printer



¹Smeenk, Wouter. "Applying automata learning to complex industrial software." Master's thesis, Radboud University Nijmegen (2012).



Literature

- ▶ Model Based Testing with Labelled Transition Systems, Jan Tretmans, 2008.
https://doi.org/10.1007/978-3-540-78917-8_1
(Main literature used for LTS in this course)
- ▶ Tester versus Bug: A Generic Framework for Model-Based Testing via Games, Petra van den Bos and Mariëlle Stoelinga, 2018. Ignore parts about games, only read: Sections 2, 2.1 and 2.3. <https://doi.org/10.48550/arXiv.1809.03098>
(This paper only considers deterministic LTS, such that definitions are slightly simpler than in this course.)
- ▶ Model-Based Testing. Mark Timmer, Ed Brinksma, Mariëlle Stoelinga (2011).
<https://doi.org/10.3233/978-1-60750-711-6-1> (Gentle introduction to MBT with slightly different notations/naming)



Labeled Transition Systems

- ▶ Labeled Transition System: (Q, L, T, q_0)
 - ▶ Q : set of states
 - ▶ L : set of labels (for actions)
 - ▶ $T \subseteq Q \times L \times Q$: transition relation
 - ▶ q_0 : initial state

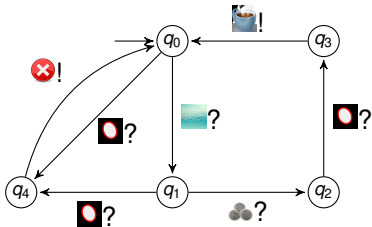
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- ▶ Labeled Transition System with inputs, outputs, and quiescence:
 (Q, L_I, L_O, T, q_0)
 - ▶ Q : set of states
 - ▶ L_I : set of input labels (for input actions, e.g. $a?$, $b?$, $c?$)
 - ▶ L_O : set of output labels (for output actions, e.g. $x!$, $y!$, $z!$)
 - ▶ Special label for quiescence: $\delta \notin L_I \cup L_O$
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Labeled Transition Systems

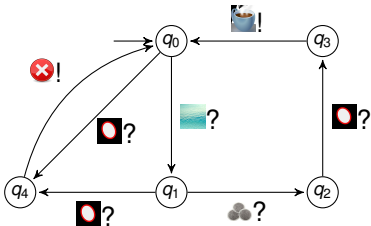
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- ▶ Note: τ out of scope for this lecture

Example coffee machine



► CM-LTS is the tuple:

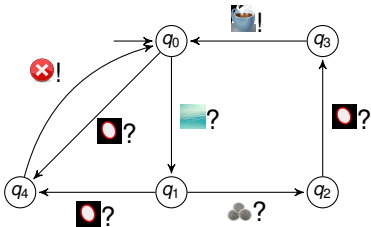
Example coffee machine



► CM-LTS is the tuple:

$$\begin{aligned} & (\{q_0, q_1, q_2, q_3, q_4\}, \\ & \{\text{cup?}, \text{bean?}, \text{cup?}\}, \\ & \{\text{cup!}, \text{X!}\}, \end{aligned}$$

Example coffee machine



► CM-LTS is the tuple:

$$\begin{aligned} & (\{q_0, q_1, q_2, q_3, q_4\}, \\ & \{\text{green rectangle?}, \text{coffee bean?}, \text{red circle?}\}, \\ & \{\text{coffee cup!}, \text{red circle!}\}, \\ & \{(q_0, \text{green rectangle?}, q_1), (q_1, \text{red circle?}, q_4), (q_4, \text{red circle!}, q_1), \dots\}, \\ & q_0) \end{aligned}$$



Quiescence

- ▶ Quiescence = absence of output
- ▶ SUT will not any produce output (until next input arrives)

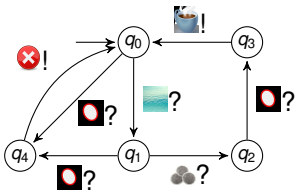


Quiescence

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 - ▶ in practice: set timeout

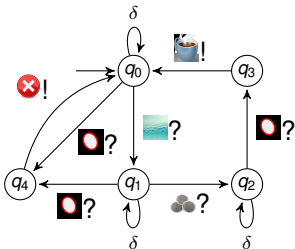
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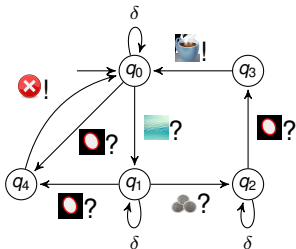
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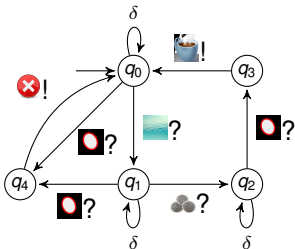
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- ▶ Some traces with quiescence (elements of $Straces(\text{CM-LTS})$):

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- ▶ Some traces with quiescence (elements of $Straces(CM-LTS)$):
 - ▶ ? δ ? ?
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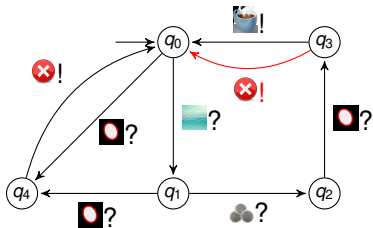
Quiescence

- ▶ Quiescence = absence of output
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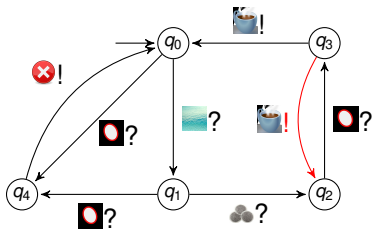
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Nondeterministic LTSs

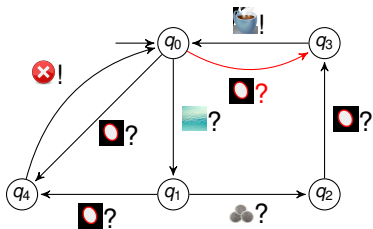


Deterministic, but no control over output

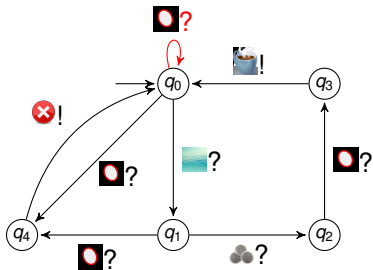
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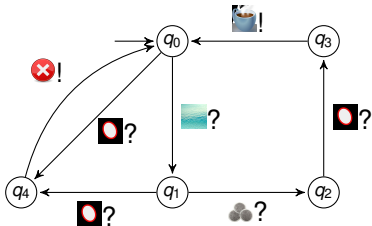
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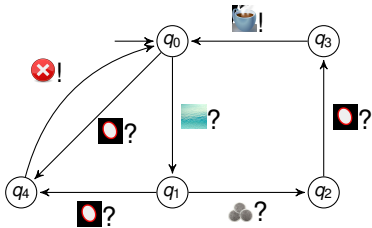
After



► Reached state after a trace:

► q_4 after $\text{❌!} =$

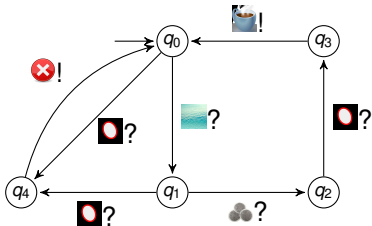
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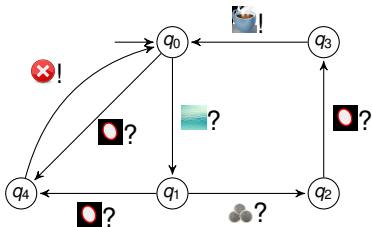
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► Reached state after a trace:

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- q_2 after 🔴? =

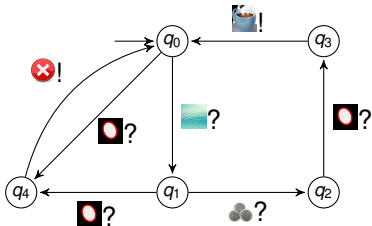
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► Reached state after a trace:

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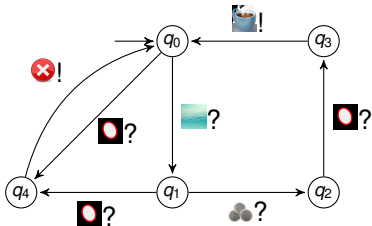
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► Reached state after a trace:

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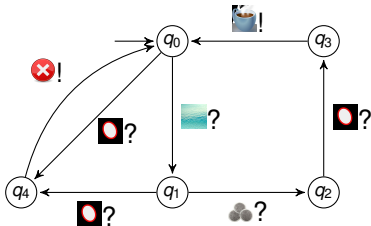
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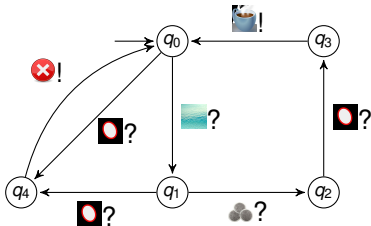
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Formal definition for *after* and *Straces*

- ▶ Let $S = (Q, L_I, L_U, T, q_0)$ be an LTS.



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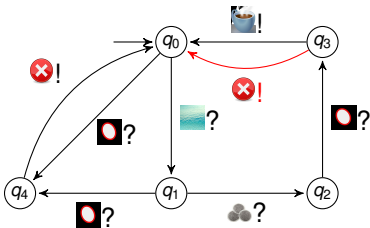
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- ▶ $Straces(q) = \{\rho' \in L^* \mid q \text{ after } \rho' \neq \emptyset\}$

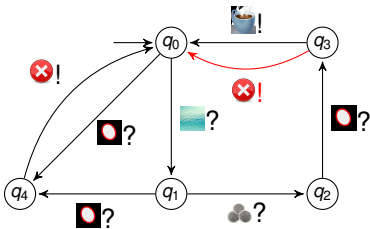
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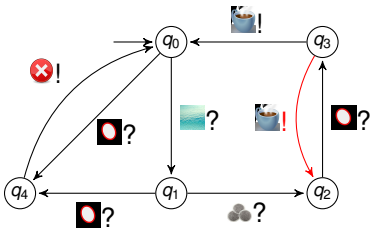
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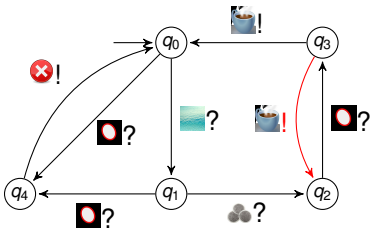
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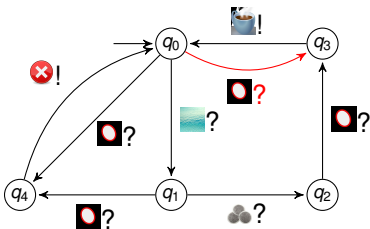
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- ▶ $q_3 \text{ after } \text{!} = \{q_0, q_2\}$



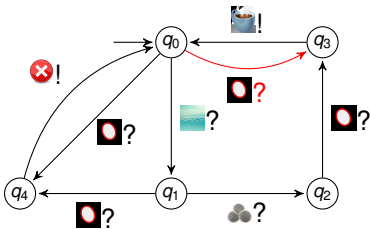
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- ▶ $q_0 \text{ after } \blacksquare? =$



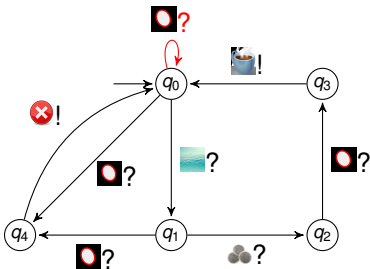
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- ▶ $q_0 \text{ after } \blacksquare? = \{q_3, q_4\}$



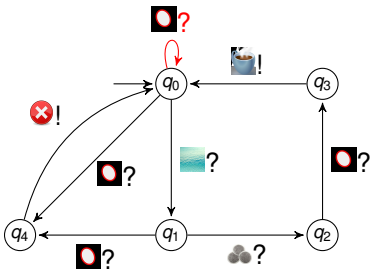
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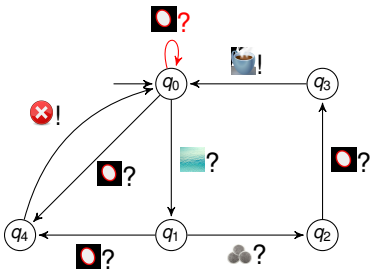
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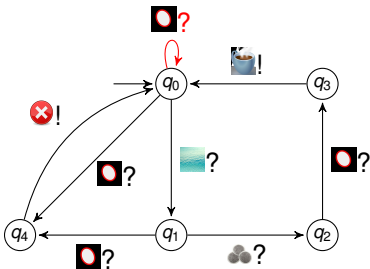
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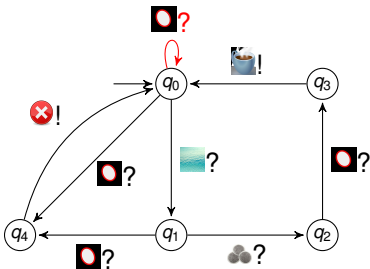
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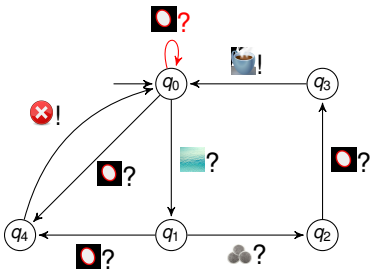
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- ▶ $Straces(q_0) = \{ \text{?}, \text{?}, \delta \text{?}, \delta \text{?}, \text{?}, \text{?}, \text{?}, \text{?}, \text{?}, \text{?}, \text{?}, \dots \}$



Formal definitions (continued)

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Formal definitions (continued)

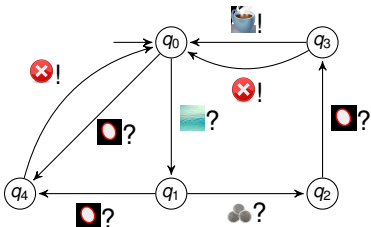
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Examples for definitions

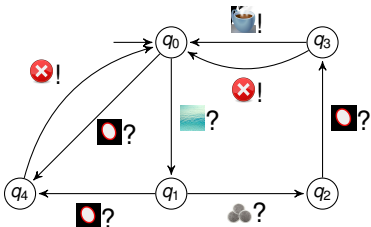
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- ▶ $out(q_3) =$

Examples for definitions

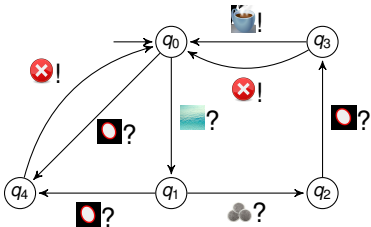
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- ▶ $out(q_3) = \{\text{blue box with white exclamation mark}, \text{red circle with white exclamation mark}\}$

Examples for definitions

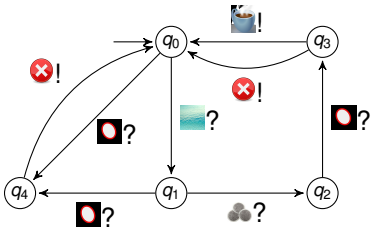
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- ▶ Deterministic?

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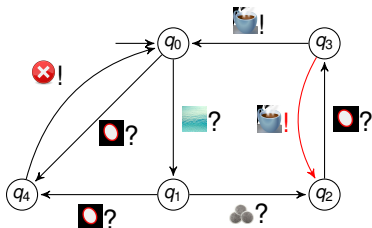
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Examples for definitions

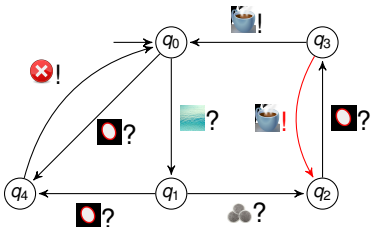
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Examples for definitions

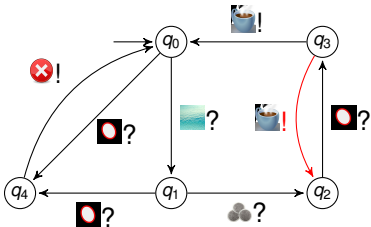
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Examples for definitions

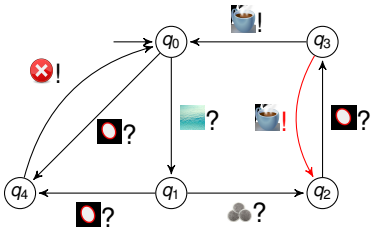
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Examples for definitions

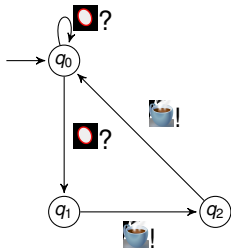
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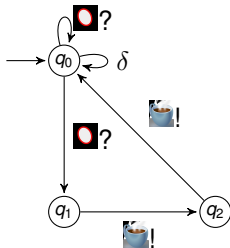
Determinisation of nondeterministic LTS

- Nondeterministic LTS:



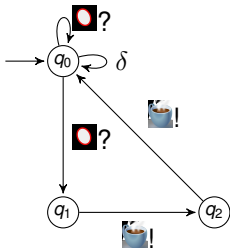
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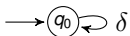


Determinisation of nondeterministic LTS

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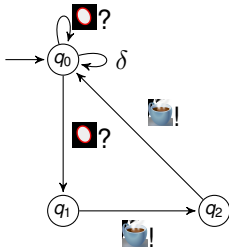


► Deterministic LTS

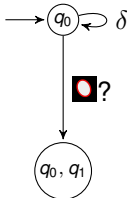


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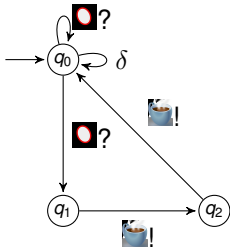


► Deterministic LTS

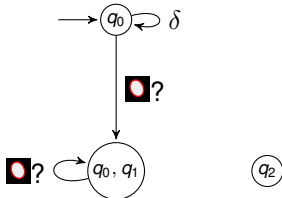


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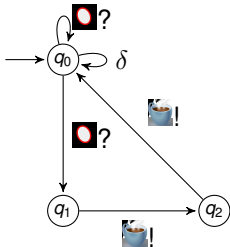


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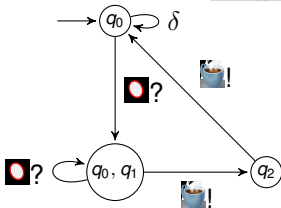


Determinisation of nondeterministic LTS

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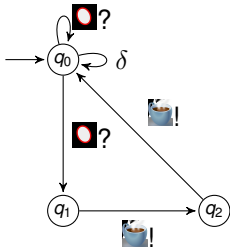


- ▶ Deterministic LTS with explicit δ

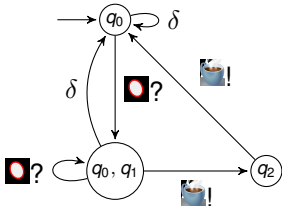


Determinisation of nondeterministic LTS

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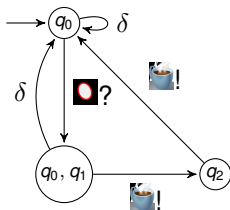


► Deterministic LTS



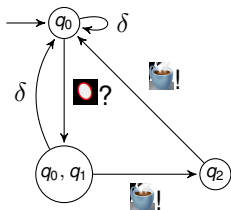
Q: does δ still denote quiescence?

Determinisation



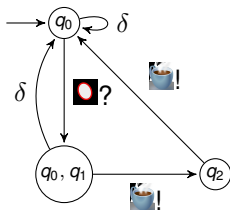
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Determinisation



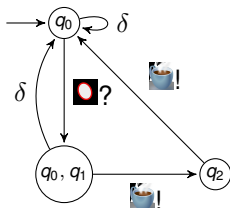
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Determinisation



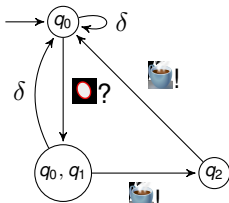
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Determinisation



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- ▶ define $after_{det}$:
 - ▶ $after_{det} : Q_{det} \times L^* \rightarrow \mathbb{P}(Q)$
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Determinisation



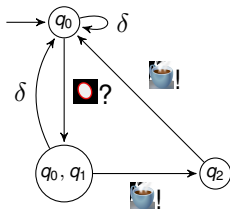
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Determinisation

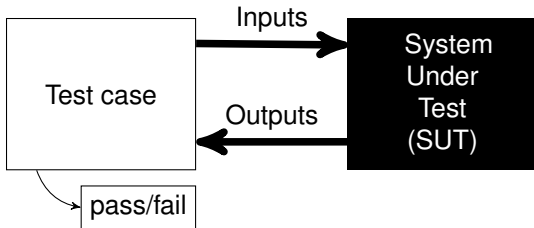


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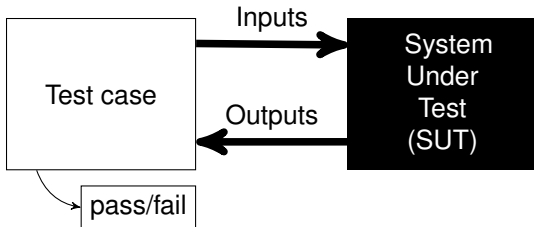
Test cases

Towards a formal definition for test cases



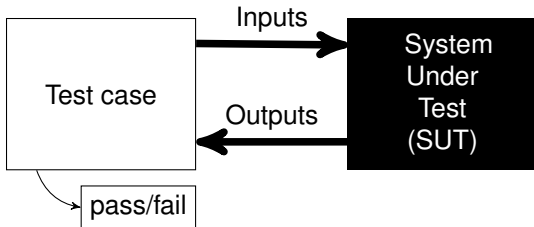
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Towards a formal definition for test cases



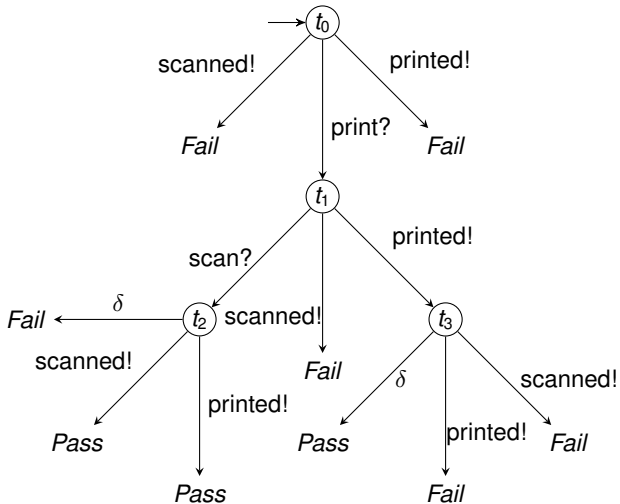
- ▶ A test case specifies:
 - ▶ which inputs should be provided to the SUT
 - ▶ which outputs are expected from the SUT
 - ▶ a verdict (pass or fail)

Towards a formal definition for test cases



- ▶ A test case specifies:
 - ▶ which inputs should be provided to the SUT
 - ▶ which outputs are expected from the SUT
 - ▶ a verdict (pass or fail)
 - ▶ a finite execution of the SUT
 - ▶ When the test case stops, it returns its verdict

Example test case for printer



Formal definition of test cases

- ▶ A test case for an LTS S is an LTS $t = (Q^t, L_I, L_U, T^t, q_0^t)$ s.t.:
 - ▶ t uses the same labels as S
 - ▶ There are two special states $Pass, Fail \in Q^t$
 - ▶ States $Pass$ and $Fail$ have self-loops for all outputs (incl. δ):
 $\forall x \in L_U \cup \{\delta\} : Pass \text{ after } x = \{Pass\}$ and $Fail \text{ after } x = \{Fail\}$
 - ▶ States $Pass$ and $Fail$ have no transition for inputs:
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 - ▶ Every state enables all outputs L_U , and either one input or δ : $\forall q \in Q^t : (|in(q)| = 0 \wedge out(q) = L_U \cup \{\delta\}) \vee (out(q) = L_U \wedge |in(q)| = 1)$

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- ▶ Self-loops omitted in graphical representations of test cases

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 - ▶ t is deterministic
 - ▶ Every state enables all outputs L_U , and either one input or δ : $\forall q \in Q^t : (|in(q)| = 0 \wedge out(q) = L_U \cup \{\delta\}) \vee (out(q) = L_U \wedge |in(q)| = 1)$

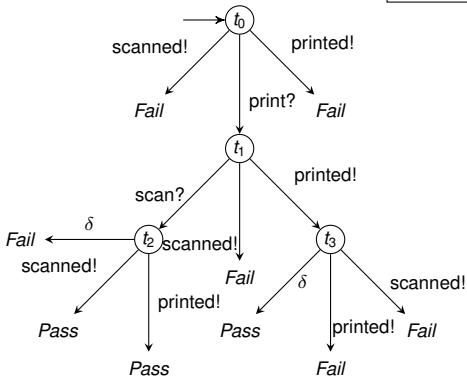
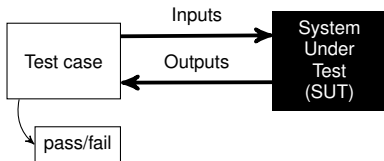
Q1: why self-loops for modelling that the test case stops?

- ▶ Self-loops omitted in graphical representations of test cases

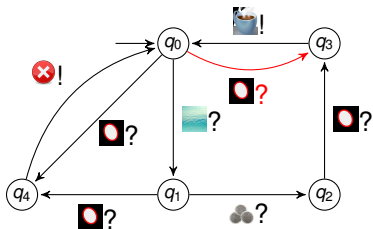
Q2: why is a test case a tree (and not a trace)?

Test execution

- ▶ Provide input, or
- ▶ wait for output, or
- ▶ stop with verdict

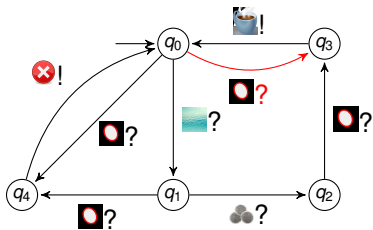


Test cases for nondeterministic LTSs

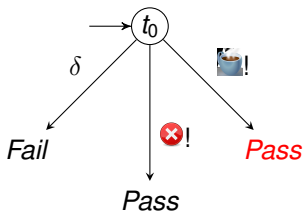


- ▶ Observe chosen -transition (after SUT executed ?)

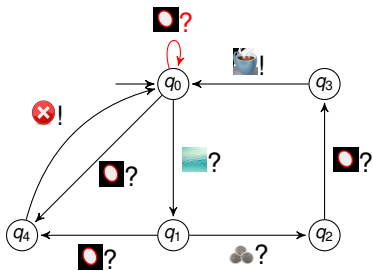
Test cases for nondeterministic LTSs



- ▶ Observe chosen **⬛?**-transition (after SUT executed **⬛?**)

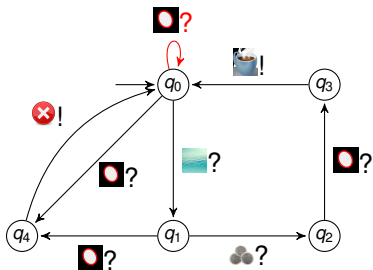


Test cases for nondeterministic LTSs

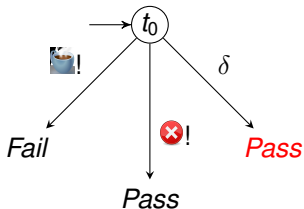


- ▶ Observe chosen -transition (after SUT executed)

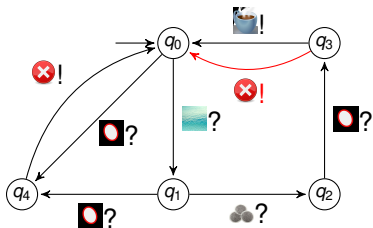
Test cases for nondeterministic LTSs



- Observe chosen -transition (after SUT executed)

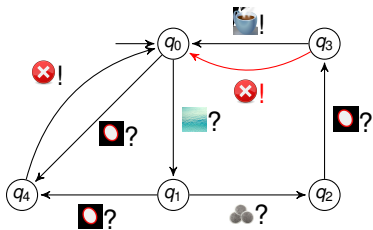


Test cases for nondeterministic LTSs

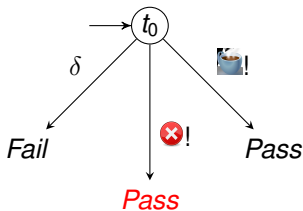


- ▶ Test case to observe , after the SUT executed   

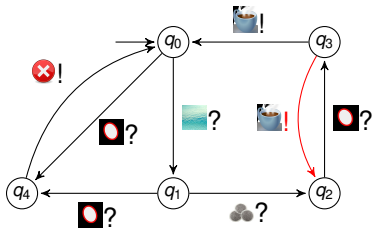
Test cases for nondeterministic LTSs



- ▶ Test case to observe $\times!$, after the SUT executed $\square? \circ? \blacksquare?$

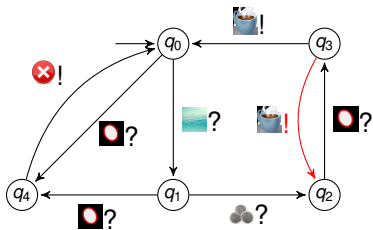


Test cases for nondeterministic LTSs

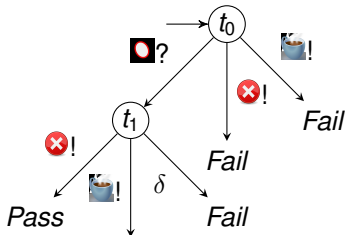


- ▶ Observe chosen -transition, after SUT executed

Test cases for nondeterministic LTSs



- Observe chosen coffee cup (!)-transition, after SUT executed blue square (?), grey circle (?), red circle with white dot (?), coffee cup (!)

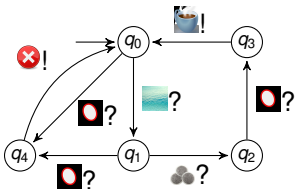




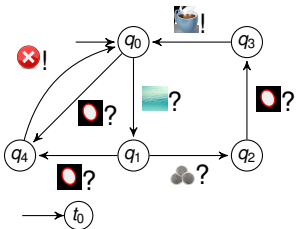
Special test cases

- ▶ Distinguishing test case: a test case that distinguishes two start states
- ▶ Homing test case: a test case that transitions to a particular state from any starting state

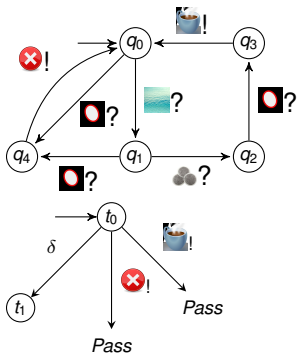
Homing test case for q_0



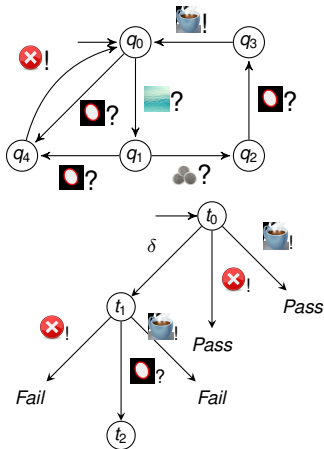
Homing test case for q_0



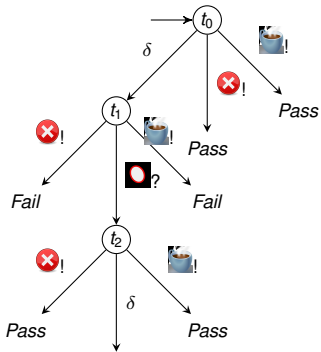
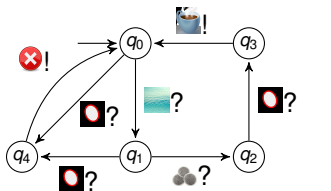
Homing test case for q_0



Homing test case for q_0



Homing test case for q_0





Next times

- ▶ Tutorial next Tuesday: exercises prepare for exam
- ▶ Lecture next Thursday: ioco & test generation algorithms