

## STAR Homework 5: test generation & ioco

**Exercise 1 (ioco and Straces)** Suppose that  $\mathcal{I} \in \text{IELTS}$  and  $\mathcal{S} \in \text{LTS}$  such that  $\mathcal{I}$  and  $\mathcal{S}$  have the same label sets.

(a) Let  $\sigma \in (L_I \cup L_U \cup \{\delta\})^*$ . Show that

$$\text{out}_{\mathcal{S}}(\sigma) = \{o \in L_U \cup \{\delta\} \mid \sigma o \in \text{Straces}_{\mathcal{S}}(s_0^{\mathcal{S}})\}.$$

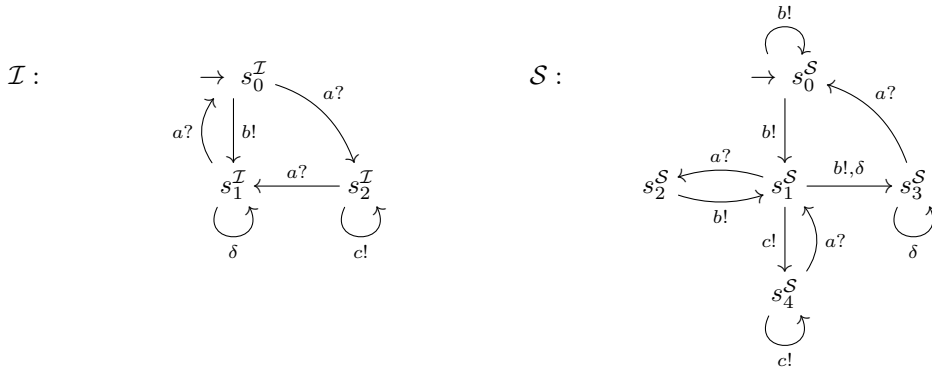
(b) Suppose that  $\text{Straces}(s_0^{\mathcal{I}}) \subseteq \text{Straces}(s_0^{\mathcal{S}})$ . Show that  $\mathcal{I} \text{ ioco } \mathcal{S}$ .

(c) Give a counter-example that shows that

$$\mathcal{I} \text{ ioco } \mathcal{S} \Rightarrow \text{Straces}(s_0^{\mathcal{I}}) \subseteq \text{Straces}(s_0^{\mathcal{S}})$$

does *not* generally hold.

**Exercise 2 (HOMEWORK: ioco in a table)** Consider the following LTSs  $\mathcal{I}$  and  $\mathcal{S}$ :



Claim: it holds that  $\mathcal{I} \text{ ioco } \mathcal{S}$ . In this exercise, you will prove this with a table-based method.

(a) Is  $\mathcal{I} \text{ ioco } \mathcal{S}$  properly defined, i.e. do  $\mathcal{I}$  and  $\mathcal{S}$  adhere to the conditions for applying **ioco**?

We need a table with two columns, plus one column for every input/output label and one column for  $\delta$ . Each column, except for the last one, will contain two sets of states, one with states of  $\mathcal{I}$  and one with states of  $\mathcal{S}$ . The first column has a row to which the initial states of  $\mathcal{I}$  and  $\mathcal{S}$  are added.

The table looks as follows:

States		$a?$		$b!$		$c!$		$\delta$		Enabled outputs	
$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$
$\{s_0^{\mathcal{I}}\}$	$\{s_0^{\mathcal{S}}\}$										

(b) Copy the table above.

(c) Fill in the  $a?$ -column for the first row of the table, as follows: under  $\mathcal{I}$ , give the set of all states that can be reached from a state in the first column via exactly one  $a?$  and any number of  $\tau$ ; in other words, the states  $s$  in  $\mathcal{I}$  such that  $s_0^{\mathcal{I}} \xrightarrow{a?} s$ . Do the same for  $\mathcal{S}$ . Note that this set may be empty.

(d) Fill in the  $b!$ -,  $c!$ -, and  $\delta$ -columns in an analogous way.

- (e) Consider all pairs of sets that you added to the  $a?$ -,  $b!$ -,  $c!$ -, and  $\delta$ -columns in (c) and (d). If both sets of a pair are non-empty *and* the pair is not yet in the first column, add the pair to the first column in a new row.
- (f) Repeat (c), (d), and (e) for an incomplete row in the table, until no incomplete rows remain.
- (g) Fill in the last column with sets of outputs (including  $\delta$ ) that are enabled in the states of  $\mathcal{I}$  and  $\mathcal{S}$  from the first column. For example:

States		$a?$		$b!$		$c!$		$\delta$		Enabled outputs	
$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$	$\mathcal{I}$	$\mathcal{S}$
$\{s_0^{\mathcal{I}}\}$	$\{s_0^{\mathcal{S}}\}$									$\{b!\}$	$\{b!\}$

- (h) Confirm that in each row of the table the enabled outputs of  $\mathcal{I}$  are a subset of the enabled outputs of  $\mathcal{S}$ . (As you will show in the next exercise, from this observation, you may conclude that  $\mathcal{I}$  **ioco**  $\mathcal{S}$ .)

**Exercise 3 (ioco in a table, proof)** We will help you prove that the table-based method from the previous exercise gives the correct outcome for *any* two QLTSs  $\mathcal{I}$ ,  $\mathcal{S}$ . The proof centers around a new QLTS  $H$ , which can be constructed from the information in the table. More precisely:

1. The states of  $H$  are the pairs of sets in the left column of the table;
  2. The initial state of  $H$  is the first row of that column;
  3. The label sets of  $H$  are the same of those of  $\mathcal{I}$  and  $\mathcal{S}$ ;
  4. There is a transition with label  $a$  in  $H$  from state  $s$  to state  $t$  if  $t$  appears in the  $s$ -row of the table.
- (a) Construct and draw the QLTS  $H$  via the method described above.
  - (b) Explain why QLTSs  $H$  obtained in this way, in general, are deterministic (i.e. each state must have at most one outgoing transition per label).
  - (c) Explain why  $H$  **after**  $\sigma = \{\langle \mathcal{I}$  **after**  $\sigma$ ,  $\mathcal{S}$  **after**  $\sigma \rangle\}$  for all  $\sigma \in \text{Straces}(s_0^H)$ .
  - (d) Explain why  $\text{Straces}(s_0^H) = \text{Straces}(s_0^{\mathcal{I}}) \cap \text{Straces}(s_0^{\mathcal{S}})$ .
  - (e) Assume that you can show that  $\text{out}(\mathcal{I}$  **after**  $\sigma) \subseteq \text{out}(\mathcal{S}$  **after**  $\sigma)$  for all  $\sigma \in \text{Straces}(s_0^{\mathcal{I}}) \cap \text{Straces}(s_0^{\mathcal{S}})$ . Prove that you can conclude that  $\mathcal{I}$  **ioco**  $\mathcal{S}$ .
  - (f) Prove that, if in each row of a constructed table the enabled outputs of  $\mathcal{I}$  are a subset of the enabled outputs of  $\mathcal{S}$ , it follows that  $\mathcal{I}$  **ioco**  $\mathcal{S}$ . Use the lemmas from (c), (d), and (e).

**Exercise 4 (Test generation)** In this exercise you generate test cases using the algorithms from the lecture.

- (a) Generate a test case for specification  $\mathcal{S}$  from exercise 2 using batch test generation. The resulting test case should be 3 levels deep, i.e. it has some Straces of length 3 to a **pass** or **fail** state.
- (b) Is your test case sound?
- (c) Are all test cases generated by the batch test generation algorithm sound?
- (d) Create an implementation  $\mathcal{I}'$  such that  $\mathcal{I}'$  **ioco**  $\mathcal{S}$  does **not** hold.
- (e) Describe how you generate and execute a test case with on-the-fly test generation and execution, such that it detects the non-conformance of  $\mathcal{I}'$  with respect to  $\mathcal{S}$ . Make sure that you need to execute a trace of length 2 or 3. If this is not possible for your implementation  $\mathcal{I}'$ , then adapt it at the previous question so that this is the case.