

# NOTATION

<b>LOGIC</b>	$p, q$	statements (or propositions)	
	$\neg p$	the negation of (statement) $p$ : <i>not</i> $p$	
	$p \wedge q$	the conjunction of $p, q$ : <i>p and</i> $q$	
	$p \vee q$	the disjunction of $p, q$ : <i>p or</i> $q$	
	$p \rightarrow q$	the implication of $q$ by $p$ : <i>p implies</i> $q$	
	$p \leftrightarrow q$	the biconditional of $p$ and $q$ : <i>p if and only if</i> $q$	
	iff	if and only if	
	$p \Rightarrow q$	logical implication: <i>p logically implies</i> $q$	
	$p \Leftrightarrow q$	logical equivalence: <i>p is logically equivalent to</i> $q$	
	$T_0$	tautology	
	$F_0$	contradiction	
	$\forall x$	For <i>all</i> $x$ (the universal quantifier)	
	$\exists x$	For <i>some</i> $x$ (the existential quantifier)	
	<b>SET THEORY</b>	$x \in A$	element $x$ is a member of set $A$
		$x \notin A$	element $x$ is not a member of set $A$
$\mathcal{U}$		the universal set	
$A \subseteq B, B \supseteq A$		$A$ is a subset of $B$	
$A \subset B, B \supset A$		$A$ is a proper subset of $B$	
$A \not\subseteq B$		$A$ is not a subset of $B$	
$A \not\subset B$		$A$ is not a proper subset of $B$	
$ A $		the cardinality, or size, of set $A$ — that is, the number of elements in $A$	
$\emptyset = \{ \}$		the empty, or null, set	
$\mathcal{P}(A)$		the power set of $A$ — that is, the collection of all subsets of $A$	
$A \cap B$		the intersection of sets $A, B$ : $\{x x \in A \text{ and } x \in B\}$	
$A \cup B$		the union of sets $A, B$ : $\{x x \in A \text{ or } x \in B\}$	
$A \Delta B$		the symmetric difference of sets $A, B$ : $\{x x \in A \text{ or } x \in B, \text{ but } x \notin A \cap B\}$	
$\bar{A}$		the complement of set $A$ : $\{x x \in \mathcal{U} \text{ and } x \notin A\}$	
$A - B$		the (relative) complement of set $B$ in set $A$ : $\{x x \in A \text{ and } x \notin B\}$	
$\bigcup_{i \in I} A_i$		$\{x x \in A_i, \text{ for at least one } i \in I\}$ , where $I$ is an index set	
$\bigcap_{i \in I} A_i$		$\{x x \in A_i, \text{ for every } i \in I\}$ , where $I$ is an index set	
<b>PROBABILITY</b>		$S$	the sample space for an experiment $\mathcal{E}$
	$A \subseteq S$	$A$ is an event	
	$Pr(A)$	the probability of event $A$	
	$Pr(A B)$	the probability of $A$ given $B$ ; conditional probability	
	$X$	random variable	
	$E(X)$	the expected value of $X$ , a random variable	
	$\text{Var}(X) = \sigma_x^2$	the variance of $X$ , a random variable	
	$\sigma_x$	the standard deviation of $X$ , a random variable	
<b>NUMBERS</b>	$a b$	$a$ divides $b$ , for $a, b \in \mathbf{Z}, a \neq 0$	
	$a \nmid b$	$a$ does not divide $b$ , for $a, b \in \mathbf{Z}, a \neq 0$	
	$\text{gcd}(a, b)$	the greatest common divisor of the integers $a, b$	
	$\text{lcm}(a, b)$	the least common multiple of the integers $a, b$	
	$\phi(n)$	Euler's phi function for $n \in \mathbf{Z}^+$	
	$[x]$	the greatest integer less than or equal to the real number $x$ : the greatest integer in $x$ : the <i>floor</i> of $x$	

## NOTATION

		$\lceil x \rceil$	the smallest integer greater than or equal to the real number $x$ : the <i>ceiling</i> of $x$
		$a \equiv b \pmod{n}$	$a$ is congruent to $b$ modulo $n$
	<b>RELATIONS</b>	$A \times B$	the Cartesian, or cross, product of sets $A, B$ : $\{(a, b)   a \in A, b \in B\}$
		$\mathcal{R} \subseteq A \times B$	$\mathcal{R}$ is a relation from $A$ to $B$
		$a \mathcal{R} b; (a, b) \in \mathcal{R}$	$a$ is related to $b$
		$a \not\mathcal{R} b; (a, b) \notin \mathcal{R}$	$a$ is not related to $b$
		$\mathcal{R}^c$	the converse of relation $\mathcal{R}$ : $(a, b) \in \mathcal{R}$ iff $(b, a) \in \mathcal{R}^c$
		$\mathcal{R} \circ \mathcal{S}$	the composite relation for $\mathcal{R} \subseteq A \times B, \mathcal{S} \subseteq B \times C$ : $(a, c) \in \mathcal{R} \circ \mathcal{S}$ if $(a, b) \in \mathcal{R}, (b, c) \in \mathcal{S}$ for some $b \in B$
		$\text{lub}\{a, b\}$	the least upper bound of $a$ and $b$
		$\text{glb}\{a, b\}$	the greatest lower bound of $a$ and $b$
		$[a]$	the equivalence class of element $a$ (relative to an equivalence relation $\mathcal{R}$ on a set $A$ ): $\{x \in A   x \mathcal{R} a\}$
	<b>FUNCTIONS</b>	$f: A \rightarrow B$	$f$ is a function from $A$ to $B$
		$f(A_1)$	for $f: A \rightarrow B$ and $A_1 \subseteq A$ , $f(A_1)$ is the image of $A_1$ under $f$ — that is, $\{f(a)   a \in A_1\}$
		$f(A)$	for $f: A \rightarrow B$ , $f(A)$ is the range of $f$
		$f: A \times A \rightarrow B$	$f$ is a binary operation on $A$
		$f: A \times A \rightarrow B (\subseteq A)$	$f$ is a closed binary operation on $A$
		$1_A: A \rightarrow A$	the identity function on $A$ : $1_A(a) = a$ for each $a \in A$
		$f _{A_1}$	the restriction of $f: A \rightarrow B$ to $A_1 \subseteq A$
		$g \circ f$	the composite function for $f: A \rightarrow B, g: B \rightarrow C$ : $(g \circ f)a = g(f(a))$ , for $a \in A$
		$f^{-1}$	the inverse of function $f$
		$f^{-1}(B_1)$	the preimage of $B_1 \subseteq B$ for $f: A \rightarrow B$
		$f \in O(g)$	$f$ is “big Oh” of $g$ ; $f$ is of order $g$
	<b>THE ALGEBRA OF STRINGS</b>	$\Sigma$	a finite set of symbols called an alphabet
		$\lambda$	the empty string
		$\ x\ $	the length of string $x$
		$\Sigma^n$	$\{x_1 x_2 \cdots x_n   x_i \in \Sigma\}, n \in \mathbf{Z}^+$
		$\Sigma^0$	$\{\lambda\}$
		$\Sigma^+$	$\bigcup_{n \in \mathbf{Z}^+} \Sigma^n$ : the set of all strings of positive length
		$\Sigma^*$	$\bigcup_{n \geq 0} \Sigma^n$ : the set of all finite strings
		$A \subseteq \Sigma^*$	$A$ is a language
		$AB$	the concatenation of languages $A, B \subseteq \Sigma^*$ : $\{ab   a \in A, b \in B\}$
		$A^n$	$\{a_1 a_2 \cdots a_n   a_i \in A \subseteq \Sigma^*\}, n \in \mathbf{Z}^+$
		$A^0$	$\{\lambda\}$
		$A^+$	$\bigcup_{n \in \mathbf{Z}^+} A^n$
		$A^*$	$\bigcup_{n \geq 0} A^n$ : the Kleene closure of language $A$
		$M = (S, \mathcal{S}, \mathcal{O}, \nu, \omega)$	a finite state machine $M$ with internal states $S$ , input alphabet $\mathcal{S}$ , output alphabet $\mathcal{O}$ , next state function $\nu: S \times \mathcal{S} \rightarrow S$ and output function $\omega: S \times \mathcal{S} \rightarrow \mathcal{O}$