

Self Study exercises Math B2, week 5

1. Find an antiderivative for each of the following functions. Check your answer by differentiating.

$$f_1(x) = 2x^{-3}$$

$$f_2(x) = -x^{-3} + x^3 - 1$$

$$f_3(x) = \frac{2}{5x}$$

$$f_4(x) = 1 + \frac{4}{3x} - \frac{1}{x^2}$$

2. Graph the function $f(x) = 1 + \sqrt{1-x^2}$ and evaluate the integral $\int_{-1}^1 f(x)dx$ with the use of areas. (See Thomas, page 319, for the definition)

3. Find the following derivatives:

(a) $\frac{d}{dx} \left(\int_1^x 3t^2 dt \right)$

(b) $\frac{d}{dx} \left(\int_1^{\sin x} 3t^2 dt \right)$

(c) $\frac{d}{dx} \left(\int_x^1 \frac{1}{t^4 + 1} dt \right)$

(d) $\frac{d}{dx} \left(\int_{\sin x}^1 \frac{1}{t^4 + 1} dt \right)$

(See Thomas, page 327, for an example)

4. (a) Graph the function $f(x) = x^3$ with $0 \leq x \leq 1$.
(b) Divide $[0, 1]$ in n subintervals with equal width. Express the Riemann sum for f on $[0, 1]$ belonging to this partition, choosing the right endpoints of the subintervals
(c) Evaluate $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k^3}{n^4}$.