Self Study exercises Math B2, week 3

1. Find and sketch the domain of $f(x, y) = \ln(x^2 + y^2 - 4)$.
   (In a sketch, axes are named and supplied with numbers, the domain is shaded; compare with Figure 14.4, Thomas, page 767).

2. The function f is defined by $f(x, y) = \sqrt{x^2 + y^2}$.

   (a) Sketch three level curves (level 0, 1 and 4) in the x-y-plane and label each level curve.

   (b) Sketch the surface $z = f(x, y)$.

3. Find the limit of $f(x, y) = \dfrac{x^2 - y^2}{x^2 + y^2}$ as $(x, y) \to (0, 0)$ or show that the limit does not exist.

4. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the following functions:

   (a) $f(x, y) = x^2 - xy + y^2$

   (b) $f(x, y) = \cos^2(3x - y^2)$

5. Find second-order partial derivatives (g_{xx}, g_{xy}, g_{yx}, g_{yy}) of the function $g(x, y) = x^2 y + \cos y + y \sin x$.
   (See Thomas, page 788, for definition and theorem.)

6. Find the equation for the tangent plane to the surface $z = \sqrt{x^2 + y^2 - 18}$ at the point $P(3, 5, 4)$.
   (See Thomas, page 811, equation (4).)