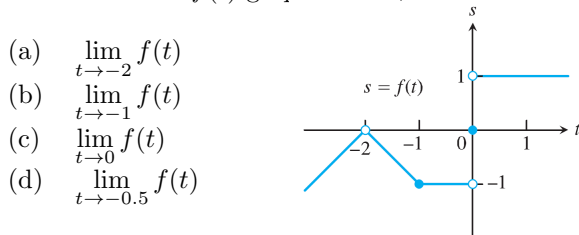


## Self Study exercises Math B2, week 1

1. For the function  $f(t)$  graphed here, find the following limits or explain why they do not exist:



(There is not a definition of 'limit' in section 2.2, but we refer to page 66 for the idea.)

2. Find  $\lim_{x \rightarrow 2} \frac{x^2 - 7x + 10}{x - 2}$  and  $\lim_{x \rightarrow 3} \frac{x^2 - 7x + 10}{x - 2}$ .

Compare to example 1, page 65, where the numerator has been factorized.

3. (a) Graph  $f(x) = \begin{cases} 1 - x^2 & \text{for } x \neq 0 \\ 2 & \text{for } x = 0 \end{cases}$

(b) Find  $\lim_{x \rightarrow 1^+} f(x)$  and  $\lim_{x \rightarrow 1^-} f(x)$ .

(c) Does  $\lim_{x \rightarrow 1} f(x)$  exist? Motivate your answer.

4. Given the function  $f(x) = \begin{cases} x^2 - 1 & \text{for } 1 \leq x < 0 \\ 2x & \text{for } 0 < x < 1 \\ 1 & \text{for } x = 1 \\ -2x + 4 & \text{for } 1 < x < 2 \\ 0 & \text{for } 2 < x < 3 \end{cases}$  graph  $f(x)$  and answer the questions (a)-(f).

(a) i. Does  $f(-1)$  exist?

ii. Does  $\lim_{x \rightarrow -1^+} f(x)$  exist?

iii. Does  $\lim_{x \rightarrow -1^-} f(x) = f(-1)$ ?

iv. Is  $f$  continuous at  $x = -1$ ? (See page 93 for the definition of continuity)

(b) i. Does  $f(1)$  exist?

ii. Does  $\lim_{x \rightarrow 1^+} f(x)$  exist?

iii. Does  $\lim_{x \rightarrow 1^-} f(x) = f(1)$ ?

iv. Is  $f$  continuous at  $x = 1$ ?

(c) i. Is  $f$  defined at  $x = 2$ ?

ii. Is  $f$  continuous at  $x = 2$ ?

(d) At what values of  $x$  is  $f$  continuous?

(e) What value should be assigned to  $f(2)$  to make the extended function continuous at  $x = 2$ ?

(f) To what new value should  $f(1)$  be changed to remove the discontinuity?

5. For the function  $f$  whose graph is given, determine the following limits:

(a)  $\lim_{x \rightarrow 4} f(x)$

(b)  $\lim_{x \rightarrow 2^+} f(x)$

(c)  $\lim_{x \rightarrow 2^-} f(x)$

(d)  $\lim_{x \rightarrow 2} f(x)$

(e)  $\lim_{x \rightarrow 3^+} f(x)$

(f)  $\lim_{x \rightarrow 3^-} f(x)$

(g)  $\lim_{x \rightarrow -3} f(x)$

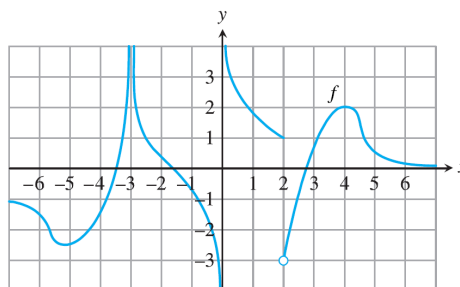
(h)  $\lim_{x \rightarrow 0^+} f(x)$

(i)  $\lim_{x \rightarrow 0^-} f(x)$

(j)  $\lim_{x \rightarrow 0} f(x)$

(k)  $\lim_{x \rightarrow \infty} f(x)$

(l)  $\lim_{x \rightarrow -\infty} f(x)$



6. Graph the rational function  $y = \frac{x^3 + 1}{x^2}$  and give the asymptotes of the graph.