Answers Linear Optimization Exercises

- This document contains only answers to check whether you solved the exercises correctly. If you need help finding complete solutions, ask the tutor.
- This year, this document is used for the second time. If you get a different answer, it is possible that the answer in this document is wrong. Please contact the tutor for any errors you find.
- Solutions of LP's with variables x and y are denoted as $(x\ y)$'
- Solutions of LP's with variables x, y and z are denoted as $(x\ y\ z)$'
- Solutions of LP's with variables $x_i, i \in \{1, \dots n\}$ are denoted as $(x_1\ x_2\ \dots x_n)$'

Session 1

1)
   b) Produce 20000/3 units of first product, 0 units of second product. Profit: 20000
   c) Yes, the investment should be made.
2)
   a) Optimum: (6 1)', objective: 5
   b) There are infinitely many optimal solutions.
3)
   a) Optimum: (6 0 0)'. Objective: 6
4)
   a) Linear
   b) Linear
   c) Linear
   d) Linear
   e) Non-Linear
   f) Linear
   g) Non-Linear
   h) Non-Linear
   i) Linear
   j) Linear
   k) Non-Linear
   l) Linear
5)
   a) Set
   b) Index
   c) Index
   d) Parameter
   e) Variable
   f) Parameter
   g) Parameter

Session 2

3)
   a) False
   b) True
   c) True
   d) False
   e) False
   f) False
   g) False

Session 3

1)
    i) Not convex, not concave
    ii) Convex, not concave
    iii) Not convex, concave
    iv) Not convex, concave
    v) Not convex, not concave
2) False
3) The proof fails
4)
    a) True
    b) True
    c) False
    d) False
    e) False
    f) False
    g) True
    h) False
    i) True
    j) False
    k) True

Session 4

2)
    a) False
    b) True
    c) True
    d) False
    e) False
    f) True
    g) False
    h) True
    i) False
    j) False
    k) False

Session 5

3)

a) $(1\ 4\ 0\ -1)'$
b) (x_1, x_2, x_3)
c)
   i) $(1\ 4\ 0\ -1)'$, $(0\ 3\ 1\ -1)'$, (-3 0 4 -1)'
   ii) $(1\ 4\ 0\ -1)'$, $(1\ 0\ 2\ -1)'$
   iii) (0 3 2 0)', (3 0 -1 -3)', (2 1 0 -2)'
d) $(0\ 3\ 2\ 0)'$
e) $(0\ 3\ 2\ 0)'$
f) 4

4)

a) False
b) False
c)
   i) False
   ii) True
   iii) False
   iv) True

6)

b) False
c) False

7)

a) True
b) False
c) False
d) True
e) False
f) False

Session 6

1)

a) $\boldsymbol{x} = (1,0,1,0,0,0)'$.
b) $\boldsymbol{d_4} = (-1,-1,0,1,0,0)', \boldsymbol{d_5} = (-2,0,-1,0,1,0)', \boldsymbol{d_6} = (0,1,0,0,0,1)'$.
c) $\overline{\boldsymbol{c}} = (0,0,0,-1,-5,1)'$
d) $\boldsymbol{d_5}, \boldsymbol{d_6}$ are feasible. $\boldsymbol{d_4}$ is not feasible.
e) Only $\boldsymbol{x} + \boldsymbol{d_6}$ is feasible.

2)

a) False
b) True
c) False
d) False
e) False
f)
   i) False
   ii) True
   iii) False

Session 7

1)
    a) Not optimal
    b) Optimal
    c) Optimal
    d) Inconclusive
2)
    a) False
    b) False
    c) False
    d) False

Session 8

1) Optimum: (4 3). Smallest number of steps: Greatest distance.
2)
    c) Unbounded
    d) (12.5 20.5)' or (28 24)'
3)
    a) True
    b) False
    c) True
    d) False
    e) True
    f) True
    g) False

Session 9

1) $B^{-1} = \begin{bmatrix} 8/5 & -4/15 & -1/3 \\ 1/5 & 2/15 & -1/3 \\ -1/5 & 1/5 & 0 \end{bmatrix}$

3)

b) $\begin{bmatrix} -15 & 0 & 2 & 0 & 0 & -3 & 10 \\ 1 & 1 & -1 & 0 & 0 & -1 & -3 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 & -2 \end{bmatrix}$

c) $(2\ 0\ 0\ 2\ 1\ 0)'$

4)

a) Use 0.5 million times process 2, and 1.5 million times process 3. Profit: $339 000 000.
b) It never changes.
c) Buy 3 million barrels, and use 8 million times process 2. Profit: $360 000 000.

5)

a) False
b) True
c) False
d) False
e) True
f) False

6)

a) False
b) True
c) False
d) False
e) True

Session 10

1) v, ii, iv, i, iii

4)

a) True
b) True
c) False
d) True
e) True
f) False
g) False

Session 11

1) You can end up with one of 2 basic feasible solutions:
   i) $x_1 = 1, x_3 = 1, x_4 = 2$

      Initial Tableau: $\begin{bmatrix} -15 & 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 & 1 \end{bmatrix}$

   ii) $x_1 = 2, x_2 = 1, x_4 = 3$

      Initial Tableau: $\begin{bmatrix} -17 & 0 & 0 & -2 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 & 1 \end{bmatrix}$

   Note that your tableau might be slightly different, depending on the order in which the variables left and entered the basis.

3)
   a) True
   b) False
   c) True
   d) True
   e) False

Session 13

5.
    a. False
    b. True
    c. True
    d. True
    e. False

Session 14

1.
    a. True
    b. False
    c. True
    d. False
    e. True
2.
    a. True
    b. True
    c. True
    d. True

Session 15

1. Let p and q denote dual variables that correspond to the first and second inequality respectively. Then in the dual optimum, $p = \frac{3}{4}, q = \frac{5}{4}$.
2.
    b. The variable corresponding to the constraint $2x \leq 5$ is 0 in the dual optimum.
    c. Systems:
        i. $p + 2q = 3, 2p + q = 2$
        ii. $p + r = 3, 2p + r = 2$
        iii. $2q + r = 3, 2p + r = 2$

    Systems i and iii yield feasible solutions.
    d. You should obtain either system i or iii.
3.
    a. True
    b. False
    c. False
    d. True
    e. False
    f. True
    g. False

Session 16

1. Shadow prices are $13 per barrel of cure oil A and $47 per barrel of crude oil B. Since we buy 3 million barrels of crude oil B at $40 in 3.21c, our profit should increase by 3 million times ($47-$40), which is exactly what happens.