Mathematical Statistics II **Winter 2023**

Assignment 4

Handed Out: Jan 12 2024 *Due: Jan 18 2024*

- Feel free to talk to other students in the class when doing this assignment. You should, however, write down your solution yourself.
- Only homeworks **submitted in the first tutorial 2024** are graded.
- Note that this homework sheet is optional, in the end we will count the best 3 out of 4 homework assignments.

Task 1. Suppose that $\{W_t\}$ and $\{Z_t\}$ are independent and identically distributed (iid) sequences, with $P(W_t = 0) = P(W_t = 1) = 0.5$ and $P(Z_t = -1) = P(Z_t = 1) = 0.5$. Define the time series model

$$X_t = W_t(1 - W_{t-1})Z_t.$$

Show that $\{X_t\}$ is white noise but not iid noise.

Task 2. Decide in the following, if the time series is a stationary process. If so, give the mean and the autocovariance function. Here, $\{W_t\}$ is i.i.d. $\mathcal{N}(0, 1)$

(a) $X_t = W_t - W_{t-3}$

(b) $X_t = W_3$

(c) $X_t = t + W_3$

(d) $X_t = W_t^2$

(e) $X_t = W_t W_{t-2}$.

Hint: You might use that $\mathbf{E}(W_t^4) = 3$.

Task 3. For a moving average process of the form

$$X_t = W_{t-1} + 2W_t + W_{t+1},$$

where W_t are independent with zero mean and variance σ_w^2, determine the autocovariance and the autocorrelation function as a function of lag τ.

Grading:

Task	1	2					3	Total
		(a)	(b)	(c)	(d)	(e)		
Points	2	1	0.5	0.5	0.5	0.5	2	7