Mathematical Statistics II		**Winter 2023**
	Assignment 1	
Handed Out: Nov 17 2023		*Due: Nov 24 2023*

- Feel free to talk to other students in the class when doing this assignment. You should, however, write down your solution yourself.
- Only homeworks **submitted in the tutorial of week 2** are graded.

Task 1: Let $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}^T = (2, -3, 1)$ and $\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}$.

(a) Find the distribution of $3X_1 - 2X_2 + X_3$.

(b) Relabel the variables if necessary and find a 2×1 vector $\mathbf{a}$ such that X_2 and $X_2 - \mathbf{a} \begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$ are independent.
Hint: Two vectors $\mathbf{Y}$ and $\mathbf{Z}$ are uncorrelated if $Cov(\mathbf{Y}, \mathbf{Z}) = 0$.

Task 2: Let $\boldsymbol{X} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$, where $\boldsymbol{\mu}^T = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 4 & 0 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 2 \end{pmatrix}$. Which of the following random variables are independent? Explain.

(a) X_1 and X_2

(b) X_1 and X_3

(c) X_2 and X_3

(d) (X_1, X_3) and X_2

(e) X_1 and $X_1 + 3X_2 - 2X_3$

Task 3: Suppose that

$$Y_i = \beta_1 X_{1i} + \beta_2 X_{2i} + \epsilon_i, \quad i = 1, \ldots, n,$$

where $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$. Notice that there is no intercept. Suppose that

$$\sum_i X_{1i} X_{2i} = 0.$$

Show that the least squares estimators $\hat{\beta}_1$ and $\hat{\beta}_2$ from the multiple regression are the same as if we were to fit separate, simple regressions on X_1 and X_2.