

## Homework assignment 5 – Mathematical Statistics

Hand in your own solutions at the start of the tutorial on October 19.

A consumer organisation is assessing the performance of a call centre. One of the quality criteria is that at least 90% of the phone calls are answered within one minute, so at most 10% of the telephone calls should be answered after more than 1 minute. A survey is conducted by observing the waiting time for  $n$  calls. To test whether the proportion  $p$  of too late answers is significantly larger than 0.1 define  $X$  as the number of waiting times exceeding one minute.

- a) Let  $p_1 > 0.1$ . Show that the test that rejects  $H_0 : p = 0.1$  against  $H_1 : p = p_1$  for large values of  $X$  is most powerful and does not depend on  $p_1$ .
- b) Using R, determine for  $n = 25$  for the aforementioned test with level of significance  $\alpha_0 = 0.05$ :
  1. The rejection region,
  2. The probability of a type I error and
  3. The power of this test if in reality 15% of the calls are answered too late.

*Hint: to compute quantiles and the c.d.f. of the binomial distribution, use the functions `qbinom()` and `pbinom()` in R.*

- c) Using R, determine for  $n = 900$  and for the test on  $H_0 : p = 0.1$  against  $H_1 : p > 0.1$  with  $\alpha_0 = 0.05$ :
  1. The rejection region,
  2. The probability of a type II error if in reality 15% of the calls are answered too late.
  3. The p-value if we observe 100 calls that are answered after 1 minute in the sample.
  4. Are the “verdicts” on the basis of the rejection region (1.) and the p-value (3.) the same?
- d) Apply a normal approximation to determine a sample size  $n$ , such that  $\alpha_0 = 0.05$  and the power for  $p = 0.15$  is at least 90%. (*you can assume that  $n$  is sufficiently large for a normal approximation, no continuity correction is needed*).

Grading:	a	b	c	d	Total
	2	1+½+1	1+1+1+1	1½	10