Homework assignment 3 – Mathematical Statistics

Hand in your own solutions at the start of the tutorial on September 28.

(a) Consider independent $X_1, \ldots, X_n \sim \text{Unif}(0, \theta)$ for unknown parameter $\theta > 0$. Show that $T(X_1, \ldots, X_n) = \max_{i=1,\ldots,n} X_i$ is a sufficient statistic.

(b) Show that the p.d.f. of $T(X_1, \ldots, X_n)$ is given by

$$x \mapsto n\theta^{-n} x^{n-1} \mathbf{1}(0 \le x \le \theta).$$

Hint: Compute first the c.d.f.

(c) Let us use now $\widehat{\theta} = \max_{i=1,\ldots,n} X_i$ as an estimator for θ (for this reason we rename T into $\widehat{\theta}$). Compute the bias, the variance and the MSE (mean squared error) of $\widehat{\theta}$.

(d) Based on the analysis in part (c), propose now an unbiased estimator. Compute the MSE of this unbiased estimator. Is it smaller than $\text{MSE}(\widehat{\theta})$?

Grading:	**a**	**b**	**c**	**d**	**Total**
	2	2	3	3	**10**